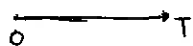


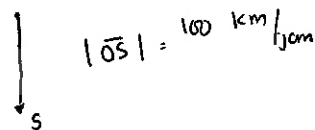
# Uji Pemahaman 1

1) a)



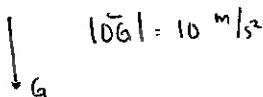
$$|\vec{T}| = 25 \text{ N}$$

b)



$$|\vec{S}| = 100 \text{ km/jam}$$

c)



$$|\vec{G}| = 10 \text{ m/s}^2$$

2)

a)



$$b) |\vec{U}| = 130 \text{ km}$$

$$\vec{U} = \begin{pmatrix} 120 \\ 50 \end{pmatrix}$$

3)

$$i) \vec{a} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$iv) \vec{PQ} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$ii) \vec{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$v) \vec{U} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$iii) \vec{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$vi) \vec{F} = \begin{pmatrix} -6 \\ -8 \end{pmatrix}$$

$$b) |\vec{a}| = 5$$

$$|\vec{PQ}| = 6$$

$$|\vec{b}| = 4$$

$$|\vec{U}| = 3$$

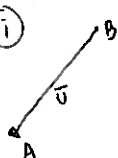
$$|\vec{c}| = 5$$

$$|\vec{F}| = 10$$

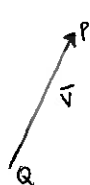
$$4) \vec{U} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$a) -\vec{U} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

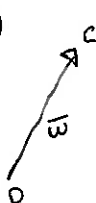
b) i)



ii)



iii)



$$c) i) \vec{U} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \vec{V} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

$$\vec{U} \neq \vec{V}$$

$$ii) |\vec{U}| = 3\sqrt{5}, |\vec{V}| = 3\sqrt{5}$$

$$|\vec{U}| = |\vec{V}|$$

$$iii) -\vec{W} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$\vec{U} = -\vec{W}$$

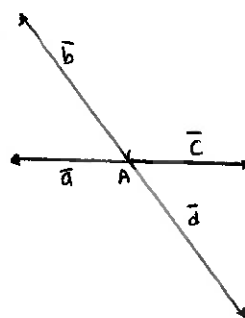
d) i)  $\vec{U}$  dan  $\vec{V}$  berlawanan

ii)  $\vec{V}$  dan  $\vec{W}$  sejajar dan searah

iii)  $\vec{U}$  dan  $-\vec{V}$  sama

iv)  $\vec{U}$  dan  $-\vec{U}$  berlawanan

5) a)



$$b) \vec{a} = -\vec{c}$$

$$\vec{b} = -\vec{d}$$

## U<sub>3</sub> Pemahaman 2

### A. Pasangkan

1)  $\vec{a} + \vec{a} = 2\vec{a}$  (A)

2)  $2\vec{a} + 3\vec{a} = 5\vec{a}$  (E)

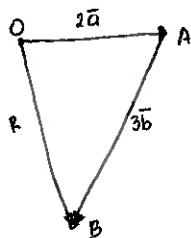
3)  $5\vec{a} - 2\vec{a} = 3\vec{a}$  (D)

4)  $4\vec{a} - 7\vec{a} = -3\vec{a}$  (C)

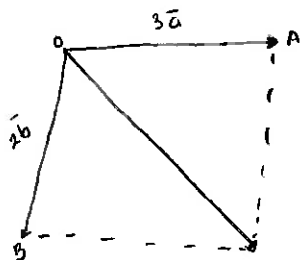
5)  $5\vec{a} - 5\vec{a} = 0$  (B)

### B. uraian

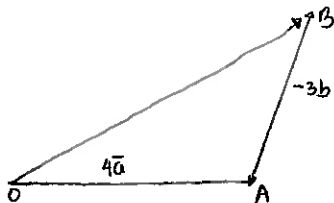
1) a)  $2\vec{a} + 3\vec{b}$



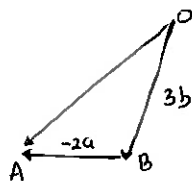
b)  $3\vec{a} + 2\vec{b}$



c)  $4\vec{a} - 3\vec{b}$



d)  $3\vec{b} - 2\vec{a}$



3) a)  $\vec{PS} = \vec{a} + \vec{b} + \vec{c}$

b)  $\vec{FS} = \frac{1}{2}(\vec{QS})$   
 $= \frac{1}{2}(\vec{b} + \vec{c})$

c)  $\vec{PF} = \vec{PA} + \vec{AF}$   
 $= \vec{a} + \frac{1}{2}(\vec{b} + \vec{c})$

d)  $\vec{FE} = \vec{FS} + \vec{SE}$   
 $= \frac{1}{2}(\vec{b} + \vec{c}) - \frac{1}{2}\vec{c}$   
 $= \frac{1}{2}\vec{b}$

e)  $\vec{PR} = \vec{a} + \vec{b}$

f)  $\vec{PE} = \vec{a} + \vec{b} + \frac{1}{2}\vec{c}$

g)  $\vec{SQ} = -(\vec{b} + \vec{c})$

4)  $3\vec{P} = 2\vec{Q}$

$$\begin{pmatrix} 3x+12y \\ 6x+3y+3 \end{pmatrix} = \begin{pmatrix} 2y-4x+4 \\ 4x-6y-2 \end{pmatrix}$$

$$\Rightarrow 3x+12y = 2y-4x+4$$

$$7x+10y = 4 \quad | \times 2 | \quad 14x+20y = 8$$

$$\Rightarrow 2x+9y = -5 \quad | \times 7 | \quad 14x+63y = -35$$

$$-43y = 43$$

$$\boxed{y = -1}$$

$$\Rightarrow 2x - 9 = -5$$

$$\boxed{x = 2}$$

5

Arah hole =  $\begin{pmatrix} 0 \\ 12 \end{pmatrix}$



2)  $\vec{OE} = \vec{OC}$   
 $\vec{E} = \vec{C} - \vec{b}$   
 $\vec{C} = \vec{b} + \vec{E}$

$$\vec{DC} = \vec{OA}$$

$$\vec{C} - \vec{d} = \vec{a}$$

$$\vec{C} = \vec{d} + \vec{a}$$

$$\begin{aligned} \Rightarrow \vec{a} + \vec{b} + \vec{c} + \vec{d} + \vec{e} \\ &= \vec{a} + \vec{c} + \vec{c} + \vec{d} \\ &= \vec{a} + 2(\vec{a} + \vec{d}) + \vec{d} \\ &= 3(\vec{d} + \vec{a}) \end{aligned}$$

A. Pilihan ganda

$$\begin{aligned} \boxed{1} \quad 2\vec{a} + \vec{b} - \vec{c} &= \begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -5 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} 7 \\ -7 \end{pmatrix} \end{aligned}$$

Jawaban : A

$$\begin{aligned} \boxed{2} \quad 3\vec{b} - (2\vec{c} - 3\vec{a}) &= \begin{pmatrix} 3 \\ 15 \end{pmatrix} - \left( \begin{pmatrix} 4 \\ 8 \end{pmatrix} - \begin{pmatrix} 9 \\ -6 \end{pmatrix} \right) \\ &= \begin{pmatrix} 3 \\ 15 \end{pmatrix} - \begin{pmatrix} -5 \\ 14 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 1 \end{pmatrix} \end{aligned}$$

$$|3\vec{b} - (2\vec{c} - 3\vec{a})| = \sqrt{64 + 1} = \sqrt{65}$$

Jawaban : A

$$\boxed{3} \quad A = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, B = \begin{pmatrix} 9 \\ -2 \end{pmatrix}, AP : PB = 3 : 4$$

$$\begin{aligned} \Rightarrow \vec{P} &= \frac{4 \begin{pmatrix} 1 \\ 5 \end{pmatrix} + 3 \begin{pmatrix} 9 \\ -2 \end{pmatrix}}{7} \\ &= \frac{\begin{pmatrix} 8 \\ 20 \end{pmatrix} + \begin{pmatrix} 27 \\ -6 \end{pmatrix}}{7} = \frac{\begin{pmatrix} 35 \\ 14 \end{pmatrix}}{7} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \end{aligned}$$

$\therefore P(5, 2)$  Jawaban : E

$$\boxed{4} \quad \vec{a} = (6, 4), \vec{b} = (m, -2), \vec{a} \perp \vec{b}$$

$$\Rightarrow \vec{b} = (-3, -2)$$

Jawaban : B

$$\begin{aligned} \boxed{5} \quad \vec{c} &= k\vec{a} + h\vec{b} \\ \begin{pmatrix} 5 \\ -12 \end{pmatrix} &= \begin{pmatrix} 3k \\ 2k \end{pmatrix} + \begin{pmatrix} 2h \\ -4h \end{pmatrix} \end{aligned}$$

$$\Rightarrow \begin{cases} 3k + 2h = 5 & \times 2 \\ 2k - 4h = -12 & \times 1 \end{cases}$$

$$\begin{aligned} \Rightarrow \begin{cases} 6k + 4h = 10 \\ 2k - 4h = -12 \end{cases} + \\ \hline 8k = -2 \\ k = -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} \Rightarrow -\frac{1}{2} - 4h &= -12 \\ \frac{23}{2} &= 4h \\ \frac{23}{8} &= h \end{aligned}$$

$$\Rightarrow k + h = -\frac{1}{4} + \frac{23}{8} = \frac{21}{8}$$

Jawaban : D

B. Uraian

$$\boxed{1} \quad \vec{a} + \vec{b} = (3, 1) + (2, 13) = (5, 12)$$

$$a) |\vec{a} + \vec{b}| = 13$$

$$b) \vec{a} + \vec{b} + \vec{c} = (5, 12) + (-2, -8) = (3, 4)$$

$$|\vec{a} + \vec{b} + \vec{c}| = 5$$

$$\boxed{2} \quad \vec{p} = 5\vec{i} + 6\vec{j}, \vec{q} = 2\vec{i} + 2\vec{j}$$

$$a) \vec{p} - \vec{q} = 3\vec{i} + 4\vec{j}$$

$$|\vec{p} - \vec{q}| = 5$$

$$b) \vec{q} - \vec{p} = -3\vec{i} - 4\vec{j}$$

$$|\vec{q} - \vec{p}| = 5$$

$$\begin{aligned} \boxed{3} \quad \vec{u} + \vec{v} + \vec{w} &= \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} -7 \\ -7 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ -4 \end{pmatrix} \end{aligned}$$

$$a) \vec{a} = \vec{u} + \vec{v} + \vec{w} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$

$$b) \text{vektor satuan } \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{-3\vec{i} - 4\vec{j}}{5}$$

$$\boxed{4} \quad \vec{a} = -2\vec{i} + 3\vec{j}, \quad \vec{b} = 3\vec{i} - 4\vec{j}$$

$$a) \quad 3\vec{a} = -6\vec{i} + 9\vec{j}$$

$$b) \quad 4\vec{b} = 12\vec{i} - 16\vec{j}$$

$$c) \quad 2\vec{a} + 3\vec{b} = (-4\vec{i} + 6\vec{j}) + (9\vec{i} - 12\vec{j}) \\ = 5\vec{i} - 6\vec{j}$$

$$d) \quad 2\vec{a} - 3\vec{b} = (-4\vec{i} + 6\vec{j}) - (9\vec{i} - 12\vec{j}) \\ = -13\vec{i} + 18\vec{j}$$

$$\boxed{5} \quad \vec{OA} = -2\vec{i} - \vec{j}, \quad \vec{OB} = 4\vec{i} + 3\vec{j}, \quad \vec{OC} = 6\vec{i}, \quad \vec{OD} = -4\vec{j}$$

$$a) \quad \vec{AB} = \vec{OB} - \vec{OA} \\ = (4\vec{i} + 3\vec{j}) - (-2\vec{i} - \vec{j}) \\ = 6\vec{i} + 4\vec{j}$$

$$b) \quad \vec{DC} = \vec{OC} - \vec{OD} \\ = 6\vec{i} + 4\vec{j}$$

$$c) \quad \vec{AD} = \vec{OD} - \vec{OA} \\ = -4\vec{j} - (-2\vec{i} - \vec{j}) \\ = 2\vec{i} - 3\vec{j}$$

$$d) \quad \vec{BC} = \vec{OC} - \vec{OB} \\ = 6\vec{i} - (4\vec{i} + 3\vec{j}) \\ = 2\vec{i} - 3\vec{j}$$

→ ABCD jajargenjang

$$\boxed{6} \quad \vec{OA} = 9\vec{i} - 10\vec{j}, \quad \vec{OB} = 4\vec{i} + 2\vec{j}, \quad \vec{OC} = m\vec{i} - 2\vec{j}$$

$$a) \quad \vec{AB} = \vec{OB} - \vec{OA} = (4\vec{i} + 2\vec{j}) - (9\vec{i} - 10\vec{j}) = -5\vec{i} + 12\vec{j}$$

$$\frac{n}{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{-5\vec{i} + 12\vec{j}}{13}$$

$$b) \quad \vec{BC} = \vec{OC} - \vec{OB} = (m\vec{i} - 2\vec{j}) - (4\vec{i} + 2\vec{j}) = (m-4)\vec{i} - 4\vec{j}$$

$$\vec{AB} = n \vec{BC}$$

$$-5\vec{i} + 12\vec{j} = n ((m-4)\vec{i} - 4\vec{j})$$

$$\rightarrow -4n = 12 \\ n = -3$$

$$\rightarrow nm - 4n = -5 \\ -3m + 12 = -5$$

$$-3m = -17$$

$$m = \frac{17}{3}$$

$$c) \quad \vec{OA} = 9\vec{i} - 10\vec{j}, \quad \vec{OB} = 4\vec{i} + 2\vec{j}, \quad \vec{OC} = \vec{i} - 2\vec{j}$$

$$\vec{OA} + k\vec{OB} + t\vec{OC} = \vec{0}$$

$$(9\vec{i} - 10\vec{j}) + (4k\vec{i} + 2k\vec{j}) + (t\vec{i} - 2t\vec{j}) = \vec{0}$$

$$(9 + 4k + t)\vec{i} + (-10 + 2k - 2t)\vec{j} = \vec{0}$$

$$\Rightarrow \begin{array}{l|l} 4k + t = -9 & \times 2 \\ 2k - 2t = 10 & \times 1 \end{array} \quad \begin{array}{l} 8k + 2t = -18 \\ 2k - 2t = 10 \end{array} +$$

$$10k = -8$$

$$k = -\frac{4}{5}$$

$$t = -9 + \frac{16}{5} = -\frac{29}{5} //$$

$$11) \quad \vec{AB} = -5\vec{i} + 12\vec{j}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = (\vec{i} - 2\vec{j}) - (4\vec{i} + 2\vec{j}) = -3\vec{i} - 4\vec{j}$$

Agar ABCD jajargenjang  $\vec{AB} = -\vec{CD}$

$$\Rightarrow \vec{CD} = -\vec{AB}$$

$$\vec{OD} - \vec{OC} = -(-5\vec{i} + 12\vec{j})$$

$$(x\vec{i} + y\vec{j}) - (\vec{i} - 2\vec{j}) = 5\vec{i} - 12\vec{j}$$

$$\Rightarrow \begin{array}{l} x - 1 = 5 \\ x = 6 \end{array} \quad \Rightarrow \begin{array}{l} y + 2 = -12 \\ y = -14 \end{array}$$

$$\vec{OD} = 6\vec{i} - 14\vec{j}$$

$$\boxed{7} \quad A = (a, -4a), \quad B = (6, -a)$$

$$\vec{AB} = \vec{B} - \vec{A} = (6, -a) - (a, -4a) = (6-a, 3a)$$

$$\Rightarrow |\vec{AB}| = \sqrt{(6-a)^2 + 9a^2} = 10$$

$$36 - 12a + a^2 + 9a^2 = 100$$

$$10a^2 - 12a - 64 = 0$$

$$2(a+2)(5a-16) = 0$$

$$a = -2 \quad \vee \quad a = \frac{16}{5}$$

$$\Rightarrow \frac{\vec{AB}}{|\vec{AB}|} = \frac{8\vec{i} - 6\vec{j}}{10} = \frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$$

8 a)  $A = (3, 2)$ ,  $B = (9, 5)$ ,  $AC : CB = 2 : 1$

$$C = \frac{\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 18 \\ 10 \end{pmatrix}}{3} = \frac{\begin{pmatrix} 21 \\ 12 \end{pmatrix}}{3} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

b)  $A = (-1, -3)$ ,  $B = (7, 5)$ ,  $AC : CB = 1 : 1$

$$C = \frac{\begin{pmatrix} -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ 5 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 6 \\ 2 \end{pmatrix}}{2} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

c)  $A = (-3, -2)$ ,  $B = (7, 3)$ ,  $AC : CB = 3 : 2$

$$C = \frac{\begin{pmatrix} -6 \\ -4 \end{pmatrix} + \begin{pmatrix} 21 \\ 9 \end{pmatrix}}{5} = \frac{\begin{pmatrix} 15 \\ 5 \end{pmatrix}}{5} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

9 a)  $P(2, 1)$ ,  $Q(4, 7)$ ,  $PR : RQ = 3 : -2$

$$R = \frac{\begin{pmatrix} -4 \\ -2 \end{pmatrix} + \begin{pmatrix} 12 \\ 21 \end{pmatrix}}{1} = \begin{pmatrix} 8 \\ 19 \end{pmatrix}$$

b)  $P(-1, -2)$ ,  $Q(4, 0)$ ,  $PR : RQ = -2 : 1$

$$R = \frac{\begin{pmatrix} -1 \\ -2 \end{pmatrix} + \begin{pmatrix} -8 \\ 0 \end{pmatrix}}{-1} = - \begin{pmatrix} -9 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

10  $P(-1, -2)$ ,  $Q(4, 3)$ ,  $R(-6, 8)$

$PA : AQ = 2 : 3$ ,  $PB : BP = 2 : 3$ ,  $QC : CR = 9 : -4$

$$\rightarrow A = \frac{3 \begin{pmatrix} -1 \\ -2 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ 3 \end{pmatrix}}{5} = \frac{\begin{pmatrix} 5 \\ 0 \end{pmatrix}}{5} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rightarrow B = \frac{3 \begin{pmatrix} -6 \\ 8 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ -2 \end{pmatrix}}{5} = \frac{\begin{pmatrix} -20 \\ 20 \end{pmatrix}}{5} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

$$\rightarrow C = \frac{-4 \begin{pmatrix} 4 \\ 3 \end{pmatrix} + 9 \begin{pmatrix} -6 \\ 8 \end{pmatrix}}{5} = \frac{\begin{pmatrix} -70 \\ 60 \end{pmatrix}}{5} = \begin{pmatrix} -14 \\ 12 \end{pmatrix}$$

$\therefore A(1, 0)$ ,  $B(-4, 4)$ ,  $C(-14, 12)$

b)  $A(1, 0)$ ,  $B(-4, 4)$ ,  $C(-14, 12)$

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (-4, 4) - (1, 0) = (-5, 4)$$

$$\overrightarrow{BC} = \overrightarrow{C} - \overrightarrow{B} = (-14, 12) - (-4, 4) = (-10, 8)$$

$$\overrightarrow{AB} = \frac{1}{2} \overrightarrow{BC} \text{ maka } A, B, C \text{ segaris}$$

# Uji Pemahaman 4

$$\begin{aligned} \text{11)} \quad 2\vec{a} + \vec{b} - \vec{c} &= \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} \end{aligned}$$

Jawaban: B

$$\begin{aligned} \text{12)} \quad 3\vec{p} - \vec{q} + 2\vec{r} &= (30\vec{i} - 9\vec{j}) - (3\vec{i} + 2\vec{j} - \vec{k}) + (8\vec{j} + 14\vec{k}) \\ &= (27\vec{i} - 3\vec{j} + 15\vec{k}) \end{aligned}$$

Jawaban: C

$$\begin{aligned} \text{13)} \quad \vec{p} \parallel \vec{q}, \vec{p} &= (-x, x, 4y+2), \vec{q} = (1-y, 3, 7x-3) \\ \Rightarrow x &= 3, y = 4 \end{aligned}$$

$$\begin{aligned} \vec{p} - \frac{1}{2}\vec{q} + \vec{r} &= (-3, 3, 18) - \left(-\frac{3}{2}, \frac{3}{2}, 9\right) + \left(\frac{3}{2}, \frac{1}{2}, 0\right) \\ &= (0, 2, 9) \rightarrow 2\vec{j} + 9\vec{k} \end{aligned}$$

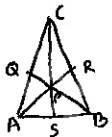
Jawaban: D

$$\begin{aligned} \text{14)} \quad \vec{AB} &= \vec{B} - \vec{A} = (3, 3, 1) - (1, 2, 3) = (2, 1, -2) \\ \vec{BC} &= \vec{C} - \vec{B} = (7, 5, -3) - (3, 3, 1) = (4, 2, -4) \end{aligned}$$

$$\Rightarrow \vec{AB} : \vec{BC} = 1 : 2$$

Jawaban: A

$$\begin{aligned} \text{15)} \quad \vec{RA} &= \vec{RC} + \vec{CA} \\ &= \frac{1}{2}(\vec{BC}) + \vec{CA} \\ &= -\frac{1}{2}\vec{v} + \vec{u} \end{aligned}$$



$$\begin{aligned} \Rightarrow \vec{PA} &= \frac{2}{3}\vec{BA} \\ &= -\frac{2}{3}\vec{v} + \frac{2}{3}\vec{u} \end{aligned}$$

$$\Rightarrow \vec{AQ} = -\frac{1}{2}\vec{v}$$

$$\begin{aligned} \Rightarrow \vec{PQ} &= \vec{PA} + \vec{AQ} \\ &= \left(-\frac{2}{3}\vec{v} + \frac{2}{3}\vec{u}\right) + \left(-\frac{1}{2}\vec{v}\right) \\ &= \frac{1}{6}\vec{u} - \frac{2}{6}\vec{v} \\ &= \frac{1}{6}\vec{u} - \frac{1}{3}\vec{v} \end{aligned}$$

Jawaban: E

## Uraian

$$\begin{aligned} \text{11)} \quad \text{a.) } \vec{AB} &= 10\vec{i} + 6\vec{j} \\ \text{b.) } \vec{CD} &= 3\vec{i} - 4\vec{j} \\ \text{c.) } \vec{PQ} &= -5\vec{i} - 5\vec{j} \\ \text{d.) } \vec{EF} &= 4\vec{i} \end{aligned}$$

$$\begin{aligned} \text{12)} \quad \text{a.) } \vec{OP} &= 2\vec{i} + 4\vec{j} + 3\vec{k} \\ \vec{OQ} &= \vec{i} - 5\vec{j} + 2\vec{k} \end{aligned}$$

$$\begin{aligned} \text{b.) } \vec{PQ} &= \vec{OQ} - \vec{OP} \\ &= -\vec{i} - 9\vec{j} - \vec{k} \end{aligned}$$

$$\begin{aligned} \text{13)} \quad \text{a.) } \vec{A} &= 4\vec{i} - \vec{j} + 2\vec{k} \\ \vec{B} &= \vec{i} + 2\vec{j} + 5\vec{k} \end{aligned}$$

$$\text{b.) } \vec{AB} = \vec{B} - \vec{A} = -3\vec{i} + 3\vec{j} + 3\vec{k}$$

$$\begin{aligned} \text{14)} \quad \text{a.) } \vec{a} + \vec{b} &= (-3, 5, 1) \end{aligned}$$

a.)

$$\text{b.) } \vec{b} + \vec{c} = (1, 7, 0)$$

$$\begin{aligned} \text{c.) } (\vec{a} + \vec{b}) + \vec{c} &= (-3, 5, 1) + (3, 4, 2) \\ &= (0, 9, 3) \end{aligned}$$

$$\begin{aligned} \vec{a} + (\vec{b} + \vec{c}) &= (-1, 2, 3) + (1, 7, 0) \\ &= (0, 9, 3) \end{aligned}$$

$$\therefore (\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

$$\text{15)} \quad \vec{PQ} = \vec{Q} - \vec{P} = (-2, -4, 6)$$

$$\vec{QR} = \vec{R} - \vec{Q} = (-1, -2, 3)$$

$$\vec{PQ} = n \cdot \vec{QR}$$

$$\vec{PQ} = 2 \cdot \vec{QR}$$

$$6) \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

karena  $\vec{a} + \vec{b} = \vec{c}$  maka  $\vec{a}, \vec{b}, \vec{c}$  membentuk segitiga

$$7) \text{vektor } \vec{I} = (2, 1, -1)$$

$$\text{vektor } \vec{II} = (4, 2, -2)$$

$$\therefore \text{vektor } \vec{II} = 2 \cdot \text{vektor } \vec{I}$$

$$\text{vektor } \vec{I} \parallel \text{vektor } \vec{II}$$

$$8) P(6, 4, 2), Q(8, 6, 4), R(2, 2, 2)$$

$$\vec{OP} = (6, 4, 2) \quad \vec{OR} = (2, 2, 2)$$

$$\vec{RQ} = (6, 4, 2) \quad \vec{PQ} = (2, 2, 2)$$

$$\text{karena } \vec{OP} = \vec{RQ}, \vec{OR} = \vec{PQ}$$

maka OPQR jajar genjang

$$9) A(2, 7, 8), B(-1, 1, -1)$$

$$\vec{AB} = \vec{B} - \vec{A} = (-3, -6, -9)$$

$$\vec{AC} = \frac{1}{3} \vec{AB} = (-1, -2, -3)$$

$$\Rightarrow \vec{AC} = -\vec{i} - 2\vec{j} - 3\vec{k} \quad \begin{aligned} \Rightarrow -1 &= x - 2 \\ 1 &= x \end{aligned}$$

$$\vec{AC} = \vec{C} - \vec{A}$$

$$\begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 7 \\ 8 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow -2 &= y - 7 \\ 5 &= y \end{aligned}$$

$$\begin{aligned} \Rightarrow -3 &= z - 8 \\ 5 &= z \end{aligned}$$

$$\therefore \vec{C} = (1, 5, 5)$$

$$10) \vec{OP} = 3\vec{i}$$

$$\vec{OR} = 4\vec{j}$$

$$\vec{OA} = 5\vec{k}$$

$$\begin{aligned} c) \vec{OD} &= \vec{OA} + \vec{AD} \\ &= 5\vec{k} + 4\vec{j} \end{aligned}$$

$$\begin{aligned} d) \vec{OD} &= \vec{OR} + \vec{RD} \\ &= -3\vec{i} + 5\vec{k} \end{aligned}$$

$$\begin{aligned} e) \vec{OC} &= \vec{OA} + \vec{AC} \\ &= 3\vec{i} + 4\vec{j} + 5\vec{k} \end{aligned}$$

$$\begin{aligned} a) \vec{PC} &= \vec{PQ} + \vec{QC} \\ &= 4\vec{j} + 5\vec{k} \end{aligned}$$

$$\begin{aligned} b) \vec{OQ} &= \vec{OR} + \vec{RQ} \\ &= 4\vec{j} + 3\vec{i} \end{aligned}$$

# Uji Pemahaman 5

## A. Pilihan ganda

$$[1] \quad \vec{p} = 3\vec{i} + 4\vec{j} + 12\vec{k}, \quad \vec{q} = 2\vec{i} + 2\vec{j} + \vec{k}$$

$$\Rightarrow \vec{p} - \vec{q} = \vec{i} + 2\vec{j} + 11\vec{k}$$

$$|\vec{p} - \vec{q}| = \sqrt{1+4+121} = \sqrt{126}$$

$$\Rightarrow \vec{p} + \vec{q} = 5\vec{i} + 6\vec{j} + 13\vec{k}$$

$$|\vec{p} + \vec{q}| = \sqrt{25+36+169} = \sqrt{230}$$

$$\begin{aligned} \Rightarrow |\vec{p}| + |\vec{q}| &= \sqrt{9+16+144} + \sqrt{4+4+1} \\ &= \sqrt{169} + \sqrt{9} \\ &= 13 + 3 \\ &= 16 \end{aligned}$$

$$\Rightarrow |\vec{p}| |\vec{q}| = 13 \times 3 = 39$$

$\therefore$  Pernyataan yang benar (i) dan (iii)

Jawaban: B

$$[2] \quad 2\vec{a} - 3\vec{b} + 4\vec{c} = \begin{pmatrix} 4 \\ -10 \\ -4 \end{pmatrix} - \begin{pmatrix} 18 \\ 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 16 \\ 12 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$$

$$\Rightarrow |2\vec{a} - 3\vec{b} + 4\vec{c}| = \sqrt{4+16+16} = \sqrt{36} = 6$$

Jawaban: D

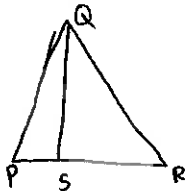
$$[3] \quad \vec{u} = \vec{a} - 2\vec{b} + 3\vec{c} = \begin{pmatrix} -7 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 6 \\ -10 \\ 8 \end{pmatrix} + \begin{pmatrix} 3 \\ -21 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -10 \\ -8 \\ -2 \end{pmatrix}$$

$$|\vec{u}| = \sqrt{100+64+4} = \sqrt{168} = 2\sqrt{42}$$

Jawaban: B

[4]



$$\Rightarrow \vec{PS} = \frac{1}{4} \vec{PR}$$

$$\Rightarrow \vec{QS} = \vec{QP} + \vec{PS}$$

$$\Rightarrow \vec{QP} = \vec{P} - \vec{Q} = (-3, 5, 3)$$

$$\vec{PR} = \vec{R} - \vec{P} = (4, -4, -4)$$

$$\Rightarrow \vec{QS} = \vec{QP} + \frac{1}{4} \vec{PR}$$

$$= (-3, 5, 3) + (1, -1, -1)$$

$$= (-2, 4, 2)$$

$$|\vec{QS}| = \sqrt{4+16+4} = \sqrt{24} = 2\sqrt{6}$$

Jawaban: B

$$[5] \quad A = (-1, -2, 3)$$

$$B = (1, 0, 9)$$

$$\vec{AB} = \vec{B} - \vec{A} = (2, -2, 6)$$

$$|\vec{AB}| = \sqrt{4+4+36} = \sqrt{44} = 2\sqrt{11}$$

Jawaban: C

## B. Uraian

$$[1] \quad a.) \sqrt{4+4+1} = \sqrt{9} = 3$$

$$b.) \sqrt{16+4+4} = \sqrt{24} = 2\sqrt{6}$$

$$c.) \sqrt{16+9} = \sqrt{25} = 5$$

$$d.) \sqrt{4+16+5} = \sqrt{25} = 5$$

$$e.) \sqrt{36+64} = \sqrt{100} = 10$$

$$f.) \sqrt{9+4+25} = \sqrt{38}$$

$$[2] \quad a.) \vec{AB} = (-7, 4, -4)$$

$$|\vec{AB}| = \sqrt{49+16+16} = \sqrt{81} = 9$$

$$b.) \vec{CD} = (8, 6, 0)$$

$$|\vec{CD}| = \sqrt{64+36} = \sqrt{100} = 10$$

$$c.) \vec{OP} = (2, 4, 4) \Rightarrow |\vec{OP}| = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$d.) |\vec{OQ}| = \sqrt{32+1+8} = \sqrt{41}$$

$$e.) \vec{PR} = (2, 1, -2)$$

$$|\vec{PR}| = \sqrt{4+1+4} = \sqrt{9} = 3$$



$$[3] \vec{a} = \vec{i} - 2\vec{j} + 2\vec{k}, \vec{b} = 3\vec{i} + 6\vec{j} - 2\vec{k}$$

$$a) |\vec{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$b) |\vec{b}| = \sqrt{9+36+4} = \sqrt{49} = 7$$

$$c) \vec{a} + \vec{b} = (4, 4, 0)$$

$$|\vec{a} + \vec{b}| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$d) \vec{a} - \vec{b} = (-2, -8, 4)$$

$$|\vec{a} - \vec{b}| = \sqrt{4+64+16} = \sqrt{84} = 2\sqrt{21}$$

$$[4] a) \vec{PQ} = \vec{Q} - \vec{P} = (4, 2, -3) - (2, -1, 3) = (2, 3, -6)$$

$$b) |\vec{PQ}| = \sqrt{4+9+36} = \sqrt{49} = 7$$

$$[5] \begin{aligned} f_1 &= (3, -5, 2) & f_3 &= (-2, 4, 1) \\ f_2 &= (1, 3, -5) & f_4 &= (-3, -2, 4) \end{aligned}$$

$$a) f_1 + f_2 + f_3 + f_4 = (-1, 0, 2)$$

$$b) |f_1| = \sqrt{9+25+4} = \sqrt{38}$$

$$|f_2| = \sqrt{1+9+25} = \sqrt{35}$$

$$|f_3| = \sqrt{4+16+1} = \sqrt{21}$$

$$|f_4| = \sqrt{9+4+16} = \sqrt{29}$$

$$|f_1 + f_2 + f_3 + f_4| = \sqrt{1+4} = \sqrt{5}$$

$$[6] f_1 = (4, 2, -3), f_2 = (0, -1, 5), f_3 = (3, 3, 2)$$

$$a) f_1 + f_2 + f_3 = (7, 4, 4)$$

$$b) |f_1 + f_2 + f_3| = \sqrt{49+16+16} = \sqrt{81} = 9$$

$$[7] \vec{AB} = (0, 2, -1) \rightarrow |\vec{AB}| = \sqrt{5}$$

$$\vec{AC} = (0, 1, -3) \rightarrow |\vec{AC}| = \sqrt{10}$$

$$\vec{BC} = (0, -1, -2) \rightarrow |\vec{BC}| = \sqrt{5}$$

$$\Rightarrow |\vec{AC}|^2 = |\vec{AB}|^2 + |\vec{BC}|^2 \quad (\text{syarat siku-siku})$$

$$10 = 5 + 5$$

$\rightarrow$  karena  $|\vec{AB}| = |\vec{BC}|$  maka  $\triangle ABC$  sama kaki

$$[8] A(x, 3, -1), B(3, 5, 0), C(-1, 4, x)$$

$$\vec{AB} = (3-x, 2, 1) \quad \vec{BC} = (-4, -1, x)$$

$$\vec{AC} = (-1-x, 1, x+1)$$

$\Rightarrow \triangle ABC$  sama kaki maka  $|\vec{AB}| = |\vec{AC}|$

$$\sqrt{(3-x)^2 + 4 + 1} = \sqrt{(-1-x)^2 + 1 + (x+1)^2}$$

$$9 - 6x + x^2 + 4 + 1 = 1 + 2x + x^2 + 1 + x^2 + 2x + 1$$

$$x^2 + 10x - 11 = 0$$

$$(x+11)(x-1) = 0$$

$$x = -11 \quad \underline{x = 1}$$

$$\Rightarrow \vec{AB} = (2, 2, 1) \quad \Rightarrow \vec{AB} + \vec{AC} + \vec{BC} = (-4, 2, 4)$$

$$\vec{AC} = (-2, 1, 2) \quad \Rightarrow \text{keliling} = |\vec{AB} + \vec{BC} + \vec{AC}|$$

$$\vec{BC} = (-4, -1, 1) \quad = \sqrt{16+4+16} = 6$$

$$[9] \vec{AB} = (4, 10, 1) \rightarrow |\vec{AB}| = \sqrt{117}$$

$$\vec{BC} = (7, 2, -8) \rightarrow |\vec{BC}| = \sqrt{117}$$

$$\vec{AD} = (-7, -2, 8) \rightarrow |\vec{AD}| = \sqrt{117}$$

$$\vec{CD} = (-4, 10, -1) \rightarrow |\vec{CD}| = \sqrt{117}$$

Karena  $|\vec{AB}| = |\vec{BC}| = |\vec{AD}| = |\vec{CD}|$  maka ABCD persegi

$$[10] A(3, -1, 5), B(4, 2, -5), C(-4, 0, 3)$$

$$\vec{AB} = (1, 3, -10)$$

$$\vec{BC} = (-8, -2, 8)$$

$$\vec{AD} = \vec{AB} + \frac{1}{2} \vec{BC}$$

$$= (1, 3, -10) + (-4, -1, 4)$$

$$= (-3, 2, -6)$$

$$|\vec{AD}| = \sqrt{9+4+36} = \sqrt{49} = 7$$



## Uji Pemahaman 6

### A. Pilihan ganda

1)  $A(2, 6, 5), B(x, y, z), P(2, 4, 5)$

$AP:PB = 2:3$

$\Rightarrow \vec{P} = \frac{3\vec{A} + 2\vec{B}}{5}$

$$\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 6 \\ 18 \\ 15 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\Rightarrow 6 + 2x = 10$

$x = 2$

$\Rightarrow 18 + 2y = 20$

$y = 1$

$\Rightarrow 15 + 2z = 25$

$z = 5$

$\Rightarrow B = (2, 1, 5)$

Jawaban: B

2)  $P(4, 3, 7), Q(-6, -7, -13), PR:RQ = 3:2$

$\Rightarrow \vec{R} = \frac{2\vec{P} + 3\vec{Q}}{5}$

$$= \frac{1}{5} \begin{pmatrix} 8 \\ 6 \\ 14 \end{pmatrix} + \frac{1}{5} \begin{pmatrix} -18 \\ -21 \\ -39 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -10 \\ -15 \\ -25 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -5 \end{pmatrix}$$

Jawaban: C

3)  $A(1, -2, -8), B(3, -4, 0), AP:PB = -3:1$

$$\Rightarrow \vec{P} = \frac{1\vec{A} + (-3)\vec{B}}{-2} = \frac{1}{-2} \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix} + \frac{3}{-2} \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 4 \end{pmatrix}$$

$\vec{P} = 4\vec{i} - 5\vec{j} + 4\vec{k}$

Jawaban: A

4)  $\vec{AB} = n \cdot \vec{BC}$

$$\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = n \begin{pmatrix} x-2 \\ y-3 \\ 3 \end{pmatrix}$$

$\Rightarrow 2 = 3n$

$\frac{2}{3} = n$

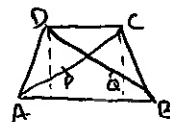
$\Rightarrow \frac{2}{3}(y-3) = 0$   
 $y = 3$

$\Rightarrow \frac{2}{3}(x-2) = 4$   
 $x-2 = 6$   
 $x = 8$

$\therefore C(8, 3, 10)$

Jawaban: D

5



$A(-6, 1, 3) \quad C(1, 4, 3)$

$B(4, 1, 3) \quad D(x, y, z)$

$\vec{AB} = (10, 0, 0)$

$\Rightarrow \vec{AD} = \vec{BC}$

$\vec{AC} = (7, 3, 0)$

$\begin{pmatrix} x+6 \\ y-1 \\ z-3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix}$

$\vec{BC} = (-3, 3, 0)$

$\Rightarrow x+6 = -3$

$y-1 = 3$

$z-3 = 0$

$x = -9$

$y = 4$

$z = 3$

$\therefore D(-9, 4, 3)$

$\Rightarrow \vec{P} = \frac{1}{2} \vec{AC}$

$= \frac{1}{2} (\vec{AB} + \vec{BC})$

$= \frac{1}{2} \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$

$\vec{Q} = \frac{1}{2} \vec{BD}$

$= \frac{1}{2} (\vec{BC} + \vec{CD})$

$= \frac{1}{2} \begin{pmatrix} -13 \\ 3 \\ 0 \end{pmatrix}$

$\Rightarrow \vec{PQ} = \vec{Q} - \vec{P}$   
 $= \begin{pmatrix} -13 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$   
 $= \begin{pmatrix} -20 \\ 0 \\ 0 \end{pmatrix}$

Jawaban: A

### B. Uraian

1) a)  $P(1, 0, 2), Q(5, 4, 10), PM:MQ = 3:1$

$\Rightarrow \vec{M} = \frac{1\vec{P} + 3\vec{Q}}{4} = \frac{1}{4} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 5 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

$\therefore M(4, 3, 8)$

b)  $P(-3, -2, -1), Q(0, -5, 2), PM:MQ = 4:-3$

$\Rightarrow \vec{M} = \frac{4\vec{P} - 3\vec{Q}}{1} = \frac{4}{1} \begin{pmatrix} -3 \\ -2 \\ -1 \end{pmatrix} - \frac{3}{1} \begin{pmatrix} 0 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ -8 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ -15 \\ 6 \end{pmatrix} = \begin{pmatrix} -12 \\ 7 \\ -10 \end{pmatrix}$

$\therefore M(-12, 7, -10)$

[2]  $A(2,5,0)$ ,  $B(5,8,3)$ ,  $C(4,7,2)$

a.)  $\overline{AB} = (3, 3, 3)$

$\overline{BC} = (-1, -1, -1)$

$\overline{AB} = -3(\overline{BC}) \Rightarrow A, B, C$  segaris

b.)  $\overline{AB} = -3 \overline{BC}$

$\frac{\overline{AB}}{\overline{BC}} = \frac{-3}{1}$

$\therefore \overline{AB} : \overline{BC} = -3 : 1$

[3]  $P(-4, 5, -6)$ ,  $Q(2, -4, 3)$

a.)  $PR : RQ = -4 : 1$

$\overline{R} = \frac{\begin{pmatrix} -4 \\ 5 \\ -6 \end{pmatrix} + \begin{pmatrix} -8 \\ 16 \\ -12 \end{pmatrix}}{-3} = \frac{\begin{pmatrix} -12 \\ 21 \\ -18 \end{pmatrix}}{-3} = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix}$

$\therefore R = (4, -7, 6)$

b.)  $PR : RQ = 4 : -1$

$\overline{R} = \frac{\begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ -16 \\ 12 \end{pmatrix}}{3} = \frac{\begin{pmatrix} 12 \\ -21 \\ 18 \end{pmatrix}}{3} = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix}$

$\therefore R = (4, -7, 6)$

[4]  $A(3, -6, 0)$ ,  $B(12, 6, -3)$

$OC : CA = 1 : 2 \Rightarrow C = \frac{1}{3} A$

$OD : DB = 1 : 2 \Rightarrow D = \frac{1}{3} B$

$\Rightarrow C = (1, -2, 0)$ ,  $D = (4, 2, -1)$

$\overline{CD} = (3, 4, -1)$

$\overline{AB} = (9, 12, -3)$

b.)  $\overline{AB} = 3 \overline{CD}$

$\frac{\overline{AB}}{\overline{CD}} = \frac{3}{1}$

$\therefore CD : AB = 1 : 3$

[5]  $A(0, 5, 4)$ ,  $B(3, 0, 1)$ ,  $C(4, 1, 0)$

$AD = \frac{1}{4} AC \Rightarrow \frac{AD}{AC} = \frac{1}{4}$

$\Rightarrow D = (x, y, z)$

$\Rightarrow x = 1$

$AD = \frac{1}{4} AC$

$\Rightarrow y - 5 = -1$

$y = 4$

$\begin{pmatrix} x \\ y-5 \\ z-4 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 4 \\ -4 \\ -4 \end{pmatrix}$

$\Rightarrow z - 4 = -1$

$z = 3$

a.)  $D = (1, 4, 3)$

$\overline{BC} = (1, 1, -1)$

$\overline{DB} = (2, -4, -2)$

b.)  $\overline{DB} \perp \overline{BC}$

$\overline{DB} \cdot \overline{BC} = 2 - 4 + 2 = 0$

karena  $\overline{DB} \cdot \overline{BC} = 0$  maka  $\overline{DB} \perp \overline{BC}$

## Uji pemahaman 7

### A. Pilihan ganda

$$[1] \theta = 60^\circ, |\vec{a}| = 4, |\vec{b}| = 3$$

$$\begin{aligned} \vec{a} \cdot (\vec{a} - \vec{b}) &= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} \\ &= |\vec{a}| |\vec{a}| - |\vec{a}| |\vec{b}| \cos 60^\circ \\ &= 16 - 12 \cdot \frac{1}{2} \\ &= 10 \end{aligned}$$

Jawaban : E

$$[2] \vec{PQ} = (3, -1, -3) = \vec{u}$$

$$\vec{QR} = (4, -4, -2) = \vec{v}$$

$$\Rightarrow \vec{u} \cdot \vec{v} = 12 + 4 + 6 = 22$$

Jawaban : D

$$\begin{aligned} [3] \vec{a} &= \vec{u} + 2\vec{v} = (3, 2, 1) + (4, 6, -2) \\ &= (7, 8, -1) \end{aligned}$$

$$\begin{aligned} \vec{b} &= 2\vec{u} - 3\vec{v} = (6, 4, 2) - (6, 9, -3) \\ &= (0, -5, 5) \end{aligned}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0 - 40 - 5 = -45$$

Jawaban : E

$$[4] \cos \alpha = \frac{8}{22}, \vec{u} = 2\vec{i} + (x+2)\vec{j} + (x-2)\vec{k}$$

$$\vec{v} = -2\vec{i} + 3\vec{j} - 3\vec{k}$$

$$\vec{u} \cdot \vec{v} = -4 + 3x + 6 - 3x + 6 = 8$$

$$|\vec{u}| = \sqrt{4 + x^2 + 4x + 4 + x^2 - 4x + 4} = \sqrt{2x^2 + 12}$$

$$|\vec{v}| = \sqrt{4 + 9 + 9} = \sqrt{22}$$

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \alpha$$

$$8 = \sqrt{2x^2 + 12} \cdot \sqrt{22} \cdot \frac{8}{22}$$

$$(22)^2 = (2x^2 + 12) \cdot 22$$

$$22 - 12 = x^2$$

$$10 = x^2$$

Jawaban : D

$$[5] \vec{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \vec{b} = \begin{pmatrix} x \\ 3 \\ 4 \end{pmatrix}, \vec{c} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$\Rightarrow (\vec{a} - \vec{b}) \cdot \vec{c} = \vec{c} \cdot \vec{a}$$

$$\vec{a} \cdot \vec{c} - \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$$

$$(2 + 0 + 3) - (2x + 3 + 12) = (2 + 0 + 3)$$

$$5 - (2x + 15) = 5$$

$$2x = -15$$

$$x = -\frac{15}{2}$$

Jawaban : B

### B. uraian

$$[1] \vec{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \vec{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$a) |\vec{i}| = 1$$

$$d) \vec{i} \cdot \vec{k} = 0$$

$$|\vec{j}| = 1$$

$$e) \vec{j} \cdot \vec{j} = 1$$

$$|\vec{k}| = 1$$

$$f) \vec{j} \cdot \vec{k} = 0$$

$$b) \vec{i} \cdot \vec{i} = 1$$

$$g) \vec{k} \cdot \vec{k} = 1$$

$$c) \vec{i} \cdot \vec{j} = 0$$

$$[2] \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$a) |\vec{a}| = 3, |\vec{b}| = 4, \theta = 60^\circ$$

$$\vec{a} \cdot \vec{b} = 3 \cdot 4 \cos 60^\circ = 6$$

$$b) |\vec{a}| = 4, |\vec{b}| = 3, \theta = 90^\circ$$

$$\vec{a} \cdot \vec{b} = 4 \cdot 3 \cos 90^\circ = 0$$

$$c) |\vec{a}| = 8, |\vec{b}| = 7, \theta = 45^\circ$$

$$\vec{a} \cdot \vec{b} = 8 \cdot 7 \cdot \cos 45^\circ = 28\sqrt{2}$$

$$d) |\vec{a}| = 2, |\vec{b}| = 1, \theta = 120^\circ$$

$$\vec{a} \cdot \vec{b} = 2 \cdot 1 \cdot \cos 120^\circ = -1$$

$$[3] \text{ a.) } \vec{a} = 2\vec{i} + 5\vec{j} + \vec{k}, \vec{b} = \vec{i} - 2\vec{j} - \vec{k}$$

$$(i) \vec{a} \cdot \vec{b} = 2 - 10 - 1 = -9$$

$$(ii) \vec{b} \cdot \vec{a} = -9$$

$$(iii) \vec{a} \cdot \vec{a} = 4 + 25 + 1 = 30$$

$$(iv) \vec{b} \cdot \vec{b} = 1 + 4 + 1 = 6$$

$$\text{b.) } \vec{a} = \vec{i} + 3\vec{j} - 2\vec{k}, \vec{b} = -\vec{i} + 2\vec{j} - 2\vec{k}$$

$$(i) \vec{a} \cdot \vec{b} = -1 + 6 + 4 = 9$$

$$(ii) \vec{b} \cdot \vec{a} = 9$$

$$(iii) \vec{a} \cdot \vec{a} = 1 + 9 + 4 = 14$$

$$(iv) \vec{b} \cdot \vec{b} = 1 + 4 + 4 = 9$$

$$[4] \vec{a} = \vec{i} + 3\vec{j} - 2\vec{k}, \vec{b} = 4\vec{i} - 2\vec{j} + 4\vec{k}$$

$$\text{a.) } \vec{a} \cdot \vec{b} = 4 - 6 - 8 = -10$$

$$\text{b.) } 3\vec{a} \cdot 2\vec{b} = (3, 9, -6) \cdot (8, -4, 8)$$

$$= 24 - 36 - 48$$

$$= -60$$

$$\text{c.) } (2\vec{a} + \vec{b}) \cdot (\vec{a} - 2\vec{b}) = (6, 4, 0) \cdot (-7, 7, -10)$$

$$= -42 + 28$$

$$= -14$$

$$\text{d.) } (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = (5, 1, 2) \cdot (-3, 5, -6)$$

$$= -15 + 5 - 12$$

$$= -22$$

a.)

$$[5] \text{ A}(1, 0, -1), \text{ B}(-2, -1, 3), \text{ C}(1, 1, 1)$$

$$\vec{a} = \vec{BA} = (3, 1, -4)$$

$$\vec{b} = \vec{BC} = (3, 2, -2)$$

$$\vec{a} \cdot \vec{b} = 9 + 2 + 8 = 19$$

$$\text{b.) } \text{A}(2, 4, 0), \text{ B}(3, 5, 1), \text{ C}(0, -4, -2)$$

$$\vec{a} = \vec{BA} = (-1, -1, -1)$$

$$\vec{b} = \vec{BC} = (-3, -9, -3)$$

$$\vec{a} \cdot \vec{b} = 3 + 9 + 3 = 15$$

# Uji Pemahaman 8

11)  $\vec{a} = (x, -1, 2)$ ,  $\vec{b} = (4, 2, -4)$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos 180^\circ$$

$$4x - 2 - 8 = \sqrt{x^2 + 5} \sqrt{36} \cdot (-1)$$

$$4x - 10 = -6\sqrt{x^2 + 5}$$

$$16x^2 - 80x + 100 = 36x^2 + 180$$

$$20x^2 + 80x + 80 = 0$$

$$20(x+1)(x+2) = 0$$

$$x = -2$$

Jawaban : E

12)  $\vec{U} = 2p\vec{i} + p\vec{j} - 6\vec{k}$ ,  $\vec{V} = p\vec{i} - 5\vec{j} + 2\vec{k}$ ,  $\vec{U} \perp \vec{V}$

$$\vec{U} \cdot \vec{V} = 0$$

$$2p^2 - 5p - 12 = 0$$

$$(2p+3)(p-4) = 0$$

$$p = -\frac{3}{2} \quad p = 4$$

Jawaban : C

13)  $\vec{a} \cdot \vec{b} = 0$

$$-k - 5 - k^3 + 3k^2 = 0$$

$$-(k+1)(k^2 - 4k + 5) = 0$$

$$k = -1$$

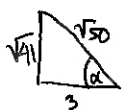
Jawaban : D

14)  $A(2, -1, 2)$ ,  $B(4, -1, 3)$ ,  $C(2, 0, 5)$

$$\vec{AB} = (2, 0, 1) \rightarrow |\vec{AB}| = \sqrt{5}$$

$$\vec{AC} = (0, 1, 3) \rightarrow |\vec{AC}| = \sqrt{10}$$

$$\cos \alpha = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{3}{\sqrt{10} \cdot \sqrt{5}} = \frac{3}{\sqrt{50}}$$



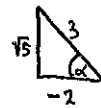
$$\tan \alpha = \frac{\sqrt{41}}{3}$$

Jawaban : D

5)  $\vec{LK} = (-1, -4, -8) \rightarrow |\vec{LK}| = \sqrt{1+16+64} = \sqrt{81} = 9$

$$\vec{LM} = (-4, 2, 4) \rightarrow |\vec{LM}| = \sqrt{16+4+16} = \sqrt{36} = 6$$

$$\cos \alpha = \frac{\vec{LK} \cdot \vec{LM}}{|\vec{LK}| |\vec{LM}|} = \frac{4 - 8 - 32}{9 \cdot 6} = \frac{-36}{54} = -\frac{2}{3}$$



$$\sin \alpha = \frac{\sqrt{5}}{3}$$

Jawaban : B

6)  $\vec{CA} = (-1, -2, 0) \rightarrow |\vec{CA}| = \sqrt{5}$

$$\vec{CB} = (5, 6, -8) \rightarrow |\vec{CB}| = \sqrt{25+36+64} = \sqrt{125} = 5\sqrt{5}$$

$$\cos C = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| |\vec{CB}|} = \frac{-5 - 12}{\sqrt{5} (5\sqrt{5})} = \frac{-17}{25}$$

Jawaban : B

7)  $\vec{AB} = (4, 2, -2) \rightarrow |\vec{AB}| = \sqrt{24} = 2\sqrt{6}$

$$\vec{AC} = (0, 0, 2) \rightarrow |\vec{AC}| = 2$$

$$\vec{BC} = (-4, -2, 4) \rightarrow |\vec{BC}| = \sqrt{36} = 6$$

Sudut terbesar : A

$$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{-4}{2\sqrt{6} \cdot 2} = -\frac{1}{\sqrt{6}}$$



$$\sin A = \frac{\sqrt{5}}{\sqrt{6}} = \frac{1}{6} \sqrt{30}$$

Jawaban : D

8)  $\vec{AB} = (2, 1) \rightarrow |\vec{AB}| = \sqrt{5}$

$$\vec{AC} = (-2, 4) \rightarrow |\vec{AC}| = \sqrt{20} = 2\sqrt{5}$$

$$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|} = \frac{-4 + 4}{10} = 0$$

$$A = 90^\circ$$

Jawaban : C

[9]  $|\vec{p}| = 5, |\vec{q}| = \sqrt{2}, |\vec{r}| = \sqrt{6}$

$\vec{r} = \vec{p} + \vec{q}, \theta = 150^\circ$

$\Rightarrow \vec{r} \cdot \vec{q} = |\vec{r}| |\vec{q}| \cos 150^\circ$

$(\vec{p} + \vec{q}) \cdot \vec{q} = \sqrt{6} \cdot \sqrt{2} \left(-\frac{1}{2}\sqrt{3}\right)$

$\vec{p} \cdot \vec{q} + \vec{q} \cdot \vec{q} = -3$

$\vec{p} \cdot \vec{q} = -3 - 2$

$= -5$

$\Rightarrow \vec{p} \cdot \vec{q} = |\vec{p}| |\vec{q}| \cos \alpha$

$-5 = 5 \cdot \sqrt{6} \cos \alpha$

$-\frac{1}{\sqrt{6}} = \cos \alpha$

$114^\circ = \alpha$

Jawaban: B

[10]  $\vec{u} = \vec{x} + \vec{y} = (-2, 0, 4) \rightarrow |\vec{u}| = \sqrt{20} = 2\sqrt{5}$

$\vec{v} = \vec{y} + \vec{z} = (-4, 0, -12) \rightarrow |\vec{v}| = \sqrt{160} = 4\sqrt{10}$

$\cos \alpha = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{8 - 48}{(2\sqrt{5})(4\sqrt{10})} = \frac{-40}{8\sqrt{50}}$

$= \frac{-40}{8 \cdot 5\sqrt{2}}$

$= -\frac{1}{2}\sqrt{2}$

$\alpha = 135^\circ$

Jawaban: E

B. uraian

[1]  $A(-2, -1, 3), B(4, 2, 3), D(3, -1, 1), AC:CB = 2:1$

$C = \frac{(-2, -1, 3) + (8, 4, 6)}{3} = \frac{(6, 3, 9)}{3} = (2, 1, 3)$

$\vec{AC} = (4, 2, 0), \vec{CD} = (1, -2, -2)$

$\vec{AC} \cdot \vec{CD} = 4 - 4 = 0$

karena  $\vec{AC} \cdot \vec{CD} = 0$  maka  $\vec{AC} \perp \vec{CD}$

$\therefore \angle ACD$  siku-siku

[2]  $\vec{A} = (4, 1, 0), \vec{B} = (-2, 3, 2), \vec{X} = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$

$\vec{C} = (0, 1, 4), \vec{D} = (2, -1, 0)$

$\Rightarrow P$  titik tengah  $AB$

$P = \left(\frac{2}{2}, \frac{4}{2}, \frac{2}{2}\right) = (1, 2, 1)$

$\Rightarrow Q$  titik tengah  $BC$

$Q = (-1, 2, 3)$

$\Rightarrow R$  titik tengah  $DC$

$R = (1, 0, 2)$

$\Rightarrow S$  titik tengah  $DA$

$S = (3, 0, 0)$

a)  $\vec{PQ} = (-2, 0, 2)$

$\vec{QR} = (2, -2, -1)$

$\vec{SR} = (-2, 0, 2)$

$\vec{PS} = (2, -2, -1)$

$\Rightarrow \vec{PQ} = \vec{SR}$  dan  $\vec{QR} = \vec{PS}$

$\therefore PQRS$  jajar genjang

b)  $\vec{PQ} \cdot \vec{QR} = |\vec{PQ}| |\vec{QR}| \cos \theta$

$-4 - 2 = \sqrt{8} \cdot \sqrt{6} \cos \theta$

$-\frac{6}{3 \cdot 2\sqrt{2}} = \cos \theta$

$-\frac{1}{2}\sqrt{2} = \cos \theta$

$135^\circ = \theta$

$\angle PQR = \angle PSR = 135^\circ$

$\angle QPS = \angle QRS = (180^\circ - 135^\circ) = 45^\circ$

$$[3] A(4, 3, -6), B(2, 1, -4), C(-1, 4, -1)$$

$$a) \cos \angle AOB = \frac{\vec{OA} \cdot \vec{OB}}{|\vec{OA}| |\vec{OB}|} = \frac{8+3+24}{\sqrt{61} \cdot \sqrt{21}} = \frac{35}{\sqrt{1281}}$$

$$ii) \cos \angle ABC = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|} = \frac{-6+6-6}{\sqrt{12} (\sqrt{27})} = \frac{-6}{18} = -\frac{1}{3}$$

$$\angle ABC = \cos^{-1} \left( -\frac{1}{3} \right)$$

$$b.) AP : PB = 3 : -1$$

$$P = \frac{\begin{pmatrix} -4 \\ -3 \\ 6 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ -12 \end{pmatrix}}{2} = \begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

$$\vec{CP} = (2, -4, -2), \vec{AB} = (-2, -2, 2)$$

$$\vec{CP} \cdot \vec{AB} = -4 + 8 - 4 = 0$$

$$\therefore \vec{CP} \perp \vec{AB}$$

$$[4] \alpha \rightarrow \angle \text{antara } AA' \text{ dan } \vec{k} \quad AA' = (5, 0, -12)$$

$$\beta \rightarrow \angle BB' \text{ dan } \vec{k} \quad BB' = (-3, 4, -12)$$

$$\gamma \rightarrow \angle CC' \text{ dan } \vec{k} \quad CC' = (-3, -4, -12)$$

$$\cos \alpha = \frac{\vec{AA'} \cdot \vec{k}}{|\vec{AA'}| |\vec{k}|} = \frac{12}{13}$$

$$\cos \beta = \frac{\vec{BB'} \cdot \vec{k}}{|\vec{BB'}| |\vec{k}|} = \frac{12}{\sqrt{9+16+144}} = \frac{12}{13}$$

$$\cos \gamma = \frac{\vec{CC'} \cdot \vec{k}}{|\vec{CC'}| |\vec{k}|} = \frac{12}{\sqrt{9+16+144}} = \frac{12}{13}$$

$$\alpha = \beta = \gamma = \cos^{-1} \left( \frac{12}{13} \right) = 22,6^\circ$$

$$[5] A = (3, 2, 1), B = (1, 2, 3)$$

$$\vec{AP} = 2(\vec{OA})$$

$$= (6, 4, 2)$$

$$\vec{BQ} = \vec{OB}$$

$$= (1, 2, 3)$$

$$\vec{OR} = (2, 4, 6)$$

$$\Rightarrow \vec{AQ} = \vec{AP} + \vec{PQ}$$

$$= (6, 4, 2) + (2, 4, 6)$$

$$= (8, 8, 8)$$

$$\therefore \vec{AQ} \parallel (\vec{i} + \vec{j} + \vec{k})$$

$$a) \vec{OC} = \vec{OA} + \vec{AC}$$

$$= (3, 2, 1) + \frac{2}{5}(\vec{AQ})$$

$$= (3, 2, 1) + \left( \frac{16}{5}, \frac{16}{5}, \frac{16}{5} \right)$$

$$= \left( \frac{31}{5}, \frac{26}{5}, \frac{21}{5} \right)$$

$$b) \vec{AB} = (-2, 0, 2)$$

$$\vec{AQ} = (8, 8, 8)$$

$$\cos \angle BAQ = \frac{\vec{AB} \cdot \vec{AQ}}{|\vec{AB}| |\vec{AQ}|} = \frac{-16 + 0 + 16}{|\vec{AB}| |\vec{AQ}|} = 0$$

$$\cos \angle BAQ = 0$$

$$\angle BAQ = 90^\circ$$



# Uji Pemahaman 9

1)  $\vec{PQ} = (3, -1, -4)$ ,  $\vec{PR} = (3, -6, 2)$

Proyeksi  $\vec{PQ}$  pada  $\vec{PR}$

$$\begin{aligned} \vec{e} &= \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PR}|^2} \cdot \vec{PR} \\ &= \frac{9 + 6 - 8}{9 + 36 + 4} \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \\ &= \frac{7}{49} (3\vec{i} - 6\vec{j} + 2\vec{k}) \\ &= \frac{1}{7} (3\vec{i} - 6\vec{j} + 2\vec{k}) \end{aligned}$$

Jawaban: C

2)  $\vec{AB} = (3, 1, -5)$ ,  $\vec{AC} = (-1, 2, -2) \rightarrow |\vec{AC}| = 3$

$$\begin{aligned} \vec{e} &= \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AC}|^2} \cdot \vec{AC} \\ &= \frac{-3 + 2 + 10}{9} \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \end{aligned}$$

Jawaban: E

3)  $\vec{a} = 2\vec{i} - 6\vec{j} - 3\vec{k}$ ,  $\vec{b} = x\vec{i} + 2\vec{j} - 4\vec{k}$

$$|\vec{e}| = \frac{4}{3}$$

$$\Rightarrow |\vec{e}| = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\frac{4}{3} = \frac{2x - 12 + 12}{\sqrt{x^2 + 20}}$$

$$4\sqrt{x^2 + 20} = 6x$$

$$16x^2 + 320 = 36x^2$$

$$320 = 20x^2$$

$$16 = x^2$$

$$\pm 4 = x$$

Jawaban: C

4)  $|\vec{e}| = \frac{2}{3}\sqrt{3}$ ,  $\vec{a} = (2, -1, p)$ ,  $\vec{b} = (p, 1, -1)$

$$|\vec{e}| = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\frac{2\sqrt{3}}{3} = \frac{2p - 1 - p}{\sqrt{p^2 + 2}}$$

$$2\sqrt{3}(\sqrt{p^2 + 2}) = 3p - 3$$

$$12(p^2 + 2) = 9p^2 - 18p + 9$$

$$12p^2 + 24 = 9p^2 - 18p + 9$$

$$3p^2 + 18p + 15 = 0$$

$$3(x+5)(x+1) = 0$$

$$x = -5 \quad x = -1$$

Jawaban: A

5)  $\vec{p} = (-3, 2, 1)$ ,  $\vec{q} = (2, 0, x)$ ,  $|\vec{e}| = \sqrt{5}$

$$\Rightarrow |\vec{e}| = \frac{\vec{p} \cdot \vec{q}}{|\vec{q}|}$$

$$\sqrt{5} = \frac{-6 + x}{\sqrt{4 + x^2}}$$

$$5(4 + x^2) = 36 - 12x + x^2$$

$$4x^2 + 12x - 16 = 0$$

$$4(x+4)(x-1) = 0$$

$$x = -4 \quad x = 1$$

Jawaban: B

6)  $\vec{P} = (2m, m^2 + 3)$ ,  $Q(3, 4)$

$$|\vec{e}| = \frac{\vec{P} \cdot \vec{Q}}{|\vec{Q}|} = \frac{6m + 4m^2 + 12}{5}$$

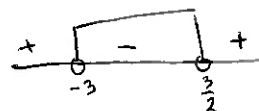
$$\Rightarrow |\vec{e}| < 6$$

$$4m^2 + 6m + 12 < 30$$

$$4m^2 + 6m - 18 < 0$$

$$2(2m-3)(m+3) < 0$$

$$m = \frac{3}{2} \quad m = -3$$



$$-3 < m < \frac{3}{2}$$

Jawaban: C

$$[7] \quad |\vec{z}| = \frac{\vec{x} \cdot \vec{y}}{|\vec{y}|} = \frac{-3+6+3}{\sqrt{3+4+9}} = \frac{6}{4} = \frac{3}{2}$$

Jawaban: C

$$[8] \quad \vec{U} = (3, -1, 1), \vec{V} = (2, p, 2) \rightarrow |\vec{V}| = \sqrt{8+p^2}$$

$$\Rightarrow |\vec{E}| = \frac{1}{2} |\vec{V}| = \frac{1}{2} \sqrt{8+p^2}$$

$$\Rightarrow |\vec{E}| = \frac{\vec{U} \cdot \vec{V}}{|\vec{V}|}$$

$$\frac{1}{2} (\sqrt{8+p^2}) = \frac{6-p+2}{\sqrt{8+p^2}}$$

$$4 + \frac{p^2}{2} = 8-p$$

$$\frac{p^2}{2} + p - 4 = 0$$

$$p^2 + 2p - 8 = 0$$

$$(p+4)(p-2) = 0$$

$$p = -4 \quad p = 2$$

Jawaban: C

$$[9] \quad \vec{U} = \vec{AB} = (3, 1, -5), |\vec{U}| = \sqrt{9+1+25} = \sqrt{35}$$

$$\vec{V} = \vec{AC} = (-1, 2, -2), |\vec{V}| = \sqrt{1+4+4} = 3$$

$$\vec{E} = \frac{\vec{U} \cdot \vec{V}}{|\vec{V}|^2} \cdot \vec{V}$$

$$= \frac{-3+2+10}{9} \cdot \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$$= -\vec{i} + 2\vec{j} - 2\vec{k}$$

Jawaban: A

$$[10] \quad \vec{U} = \vec{AB} = (-3, 4, -1)$$

$$\vec{V} = \vec{AC} = (2, -4, -4) \rightarrow |\vec{V}| = \sqrt{36} = 6$$

$$\vec{E} = \frac{\vec{U} \cdot \vec{V}}{|\vec{V}|^2} \cdot \vec{V}$$

$$= \frac{-6-16+4}{36} \cdot \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix}$$

$$= -\frac{18}{36} \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$$

$$\therefore \vec{E} = -\vec{i} + 2\vec{j} + 2\vec{k}$$

Jawaban: C

## B. Uraian

$$[1] \quad P(2, 4, 3), Q(1, -5, 2)$$

a) Panjang proyeksi  $\vec{P}$  ke  $\vec{Q}$

$$|\vec{E}| = \frac{\vec{P} \cdot \vec{Q}}{|\vec{Q}|} = \frac{2-20+6}{\sqrt{1+25+4}} = \frac{-12}{\sqrt{30}} = -\frac{12}{\sqrt{30}}$$

$$\vec{E} = \frac{\vec{P} \cdot \vec{Q}}{|\vec{Q}|^2} \cdot \vec{Q}$$

$$= -\frac{12}{30} \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{4}{5} \\ 2 \\ -\frac{4}{5} \end{pmatrix}$$

$$\text{vektor proyeksi } \vec{P} \text{ ke } \vec{Q} = -\frac{2}{5}\vec{i} + 2\vec{j} - \frac{4}{5}\vec{k}$$

b) Panjang proyeksi  $\vec{Q}$  ke  $\vec{P}$

$$|\vec{E}| = \frac{\vec{Q} \cdot \vec{P}}{|\vec{P}|} = \frac{2-20+6}{\sqrt{4+16+9}} = \frac{-12}{\sqrt{29}} = -\frac{12}{\sqrt{29}}$$

$$\vec{E} = \frac{\vec{Q} \cdot \vec{P}}{|\vec{P}|^2} \cdot \vec{P} = -\frac{12}{29} \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -\frac{24}{29} \\ -\frac{48}{29} \\ -\frac{36}{29} \end{pmatrix}$$

$$\text{vektor proyeksi } \vec{Q} \text{ ke } \vec{P} = -\frac{24}{29}\vec{i} - \frac{48}{29}\vec{j} - \frac{36}{29}\vec{k}$$

$$[2] \quad \vec{P} = 7\vec{i} + 4\vec{j} - 4\vec{k}, \vec{Q} = \vec{i} + \vec{j} - 2\vec{k}$$

$$\text{a) vektor satuan } \vec{P} = \frac{\vec{P}}{|\vec{P}|} = \frac{1}{9} \begin{pmatrix} 7 \\ 4 \\ -4 \end{pmatrix}$$

$$\text{vektor satuan } \vec{P} = \frac{7}{9}\vec{i} + \frac{4}{9}\vec{j} - \frac{4}{9}\vec{k}$$

$$\text{vektor satuan } \vec{Q} = \frac{\vec{Q}}{|\vec{Q}|} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\text{vektor satuan } \vec{Q} = \frac{\sqrt{6}}{6}\vec{i} + \frac{\sqrt{6}}{6}\vec{j} - \frac{2}{6}\sqrt{6}\vec{k}$$

$$[2] \quad p = 7\bar{i} + 4\bar{j} - 4\bar{k}, \quad q = \bar{i} + \bar{j} - 2\bar{k}$$

$$|p| = 9, \quad |q| = \sqrt{6}$$

b.) ~~per~~ misal  $\bar{e}$  = proyeksi  $\bar{p}$  ke  $\bar{q}$

$$|\bar{e}| = \frac{\bar{p} \cdot \bar{q}}{|q|} = \frac{7+4+8}{\sqrt{6}} = \frac{19}{\sqrt{6}}$$

$$\bar{e} = \frac{\bar{p} \cdot \bar{q}}{|q|^2} \bar{q} = \frac{19}{6} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\bar{e} = \frac{19}{6} \bar{i} + \frac{19}{6} \bar{j} - \frac{19}{3} \bar{k}$$

c.) misal  $\bar{c}$  proyeksi  $\bar{q}$  ke  $\bar{p}$

$$|\bar{c}| = \frac{\bar{q} \cdot \bar{p}}{|p|} = \frac{19}{9}$$

$$\bar{c} = \frac{\bar{q} \cdot \bar{p}}{|p|^2} \bar{p} = \frac{19}{81} \begin{pmatrix} 7 \\ 4 \\ -4 \end{pmatrix}$$

$$\bar{c} = \frac{133}{81} \bar{i} + \frac{76}{81} \bar{j} - \frac{76}{81} \bar{k}$$

$$[3] \quad \overline{pq} = (-7, -4, 4) \rightarrow |\overline{pq}| = 9$$

$$\bar{a} = -3\bar{i} + 4\bar{j} + \bar{k} \rightarrow |\bar{a}| = \sqrt{26}$$

a.) proyeksi  $\bar{a}$  pada  $\overline{pq}$

$$|\bar{e}| = \frac{\bar{a} \cdot \overline{pq}}{|\overline{pq}|} = \frac{21-16+4}{9} = \frac{9}{9} = 1$$

b.) proyeksi  $\overline{pq}$  pada  $\bar{a}$

$$|\bar{e}| = \frac{\overline{pq} \cdot \bar{a}}{|\bar{a}|} = \frac{9}{\sqrt{26}} = \frac{9}{26} \sqrt{26}$$

$$\bar{e} = \frac{\overline{pq} \cdot \bar{a}}{|\bar{a}|^2} \bar{a} = \frac{9}{26} \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}$$

$$\bar{e} = -\frac{27}{26} \bar{i} + \frac{18}{13} \bar{j} + \frac{9}{26} \bar{k}$$

$$[4] \quad p = (-6, 2, -3), \quad q(1, 2, -2)$$

a.) vektor proyeksi  $\bar{p}$  pada  $\bar{q}$

$$\bar{e} = \frac{\bar{p} \cdot \bar{q}}{|\bar{q}|^2} \bar{q} = \frac{-6+4+6}{1+4+4} \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \frac{4}{9} \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$\bar{e} = \frac{4}{9} \bar{i} + \frac{8}{9} \bar{j} - \frac{8}{9} \bar{k}$$

b.) vektor proyeksi  $\bar{q}$  pada  $\bar{p}$

$$\bar{e} = \frac{\bar{q} \cdot \bar{p}}{|\bar{p}|^2} \bar{p} = \frac{4}{7} \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix}$$

$$\bar{e} = -\frac{24}{7} \bar{i} + \frac{8}{7} \bar{j} - \frac{12}{7} \bar{k}$$

$$c.) \quad \bar{p} + \bar{q} = (-5, 4, -5) \rightarrow |\bar{p} + \bar{q}| = \sqrt{66}$$

$$\bar{p} - \bar{q} = (-7, 0, -1) \rightarrow |\bar{p} - \bar{q}| = \sqrt{50}$$

d.) vektor proyeksi  $\bar{p} + \bar{q}$  pada  $\bar{p} - \bar{q}$

$$\bar{e} = \frac{(\bar{p} + \bar{q}) \cdot (\bar{p} - \bar{q})}{|\bar{p} - \bar{q}|^2} (\bar{p} - \bar{q})$$

$$= \frac{35+5}{50} \begin{pmatrix} -7 \\ 0 \\ -1 \end{pmatrix} = \frac{4}{5} \begin{pmatrix} -7 \\ 0 \\ -1 \end{pmatrix}$$

$$\bar{e} = -\frac{28}{5} \bar{i} - \frac{4}{5} \bar{k}$$

$$[5] \quad A = \bar{a} = (1, 1, 7), \quad B = \bar{b} = (7, -5, 1)$$

$$AC : CB = 5 : 1$$

$$\bar{c} = \frac{(1, 1, 7) + (35, -25, 5)}{6}$$

$$= \frac{(36, -24, 12)}{6}$$

$$= (6, -4, 2)$$

$$\boxed{15} \quad \vec{a} = (1, 1, 7) \rightarrow |\vec{a}| = \sqrt{51}$$

$$\vec{b} = (7, -5, 1) \rightarrow |\vec{b}| = \sqrt{75} = 5\sqrt{3}$$

$$\vec{c} = (6, -4, 2) \rightarrow |\vec{c}| = \sqrt{56} = 2\sqrt{14}$$

$\Rightarrow$  Panjang vektor proyeksi  $\vec{a}$  tndp  $\vec{c}$  ( $\vec{e}$ )

$$|\vec{e}| = \frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} = \frac{6 - 4 + 14}{2\sqrt{14}} = \frac{16}{2\sqrt{14}} = \frac{8}{\sqrt{14}} \sqrt{14}$$

$$= \frac{4}{7} \sqrt{14}$$

$\Rightarrow$  Panjang proyeksi  $\vec{c}$  tndp  $\vec{b}$  ( $\vec{f}$ )

$$|\vec{f}| = \frac{\vec{c} \cdot \vec{b}}{|\vec{b}|} = \frac{42 + 20 + 2}{5\sqrt{3}} = \frac{64}{5\sqrt{3}} = \frac{64}{15} \sqrt{3}$$

# Soal Model AKM

misalkan posisi kunyit = a, lengkuas = b

jahne = c, lidah buaya = d, temulawak = e

kencur = f, kemangi = g, kumis kucing = h

$$\vec{a} = (-1, 7) \rightarrow |\vec{a}| = \sqrt{50} = 5\sqrt{2}$$

$$\vec{b} = (2, 5) \rightarrow |\vec{b}| = \sqrt{29}$$

$$\vec{c} = (-3, 4) \rightarrow |\vec{c}| = 5$$

$$\vec{d} = (1, 4) \rightarrow |\vec{d}| = \sqrt{17}$$

$$\vec{e} = (4, 3) \rightarrow |\vec{e}| = 5$$

$$\vec{f} = (-4, 1) \rightarrow |\vec{f}| = \sqrt{17}$$

$$\vec{g} = (-4, -2) \rightarrow |\vec{g}| = \sqrt{20} = 2\sqrt{5}$$

$$\vec{h} = (-2, -3) \rightarrow |\vec{h}| = \sqrt{13}$$

$$\text{1) (i) } \vec{f} \cdot \vec{d} = -4 + 4 = 0$$

$$\vec{f} \perp \vec{d} \quad (\text{Benar})$$

$$\text{(ii) } |\vec{e}| = |\vec{c}| \quad (\text{Benar})$$

(iii) Salah

(iv) Benar

(v) Salah

$$\text{2) } \left. \begin{array}{l} |\vec{h}| = \sqrt{13} \\ |\vec{f}| = \sqrt{17} \end{array} \right\} |\vec{h}| < |\vec{f}|$$

(i) Benar

(ii) Salah

(iii) Salah

$$\begin{aligned} \cos \angle \vec{g} \vec{h} &= \frac{\vec{g} \cdot \vec{h}}{|\vec{g}| |\vec{h}|} = \frac{8 + 6}{2\sqrt{5} \cdot \sqrt{13}} = \frac{14}{2\sqrt{65}} \\ &= \frac{7}{\sqrt{65}} \end{aligned}$$

(iv) Benar

$$\cos \angle \vec{d} \vec{e} = \frac{\vec{d} \cdot \vec{e}}{|\vec{d}| |\vec{e}|} = \frac{4 + 12}{\sqrt{17} \cdot 5} = \frac{16}{5\sqrt{17}} = \frac{16}{5 \cdot 17} \sqrt{17} = \frac{16}{85} \sqrt{17}$$

(v) Benar

$$\text{3) kencur ke jahne: } \vec{f} \vec{c} = (1, 3)$$

$$\text{kunyit ke lidah buaya: } \vec{a} \vec{d} = (2, -3)$$

$$|\vec{f} \vec{c}| = \sqrt{10}, |\vec{a} \vec{d}| = \sqrt{13}$$

$$\Rightarrow |\vec{a} \vec{d}| - |\vec{f} \vec{c}| = \sqrt{13} - \sqrt{10}$$

Jawaban: B

# Latihan Soal akhir Bab 7

11  $A(-2, 2, -5) ; B(3, -8, 5) ; C(-1, -3, 0)$

$$\Rightarrow \vec{AB} = \vec{B} - \vec{A} \quad \Rightarrow AQ : QB = 3 : 2$$

$$= \begin{pmatrix} 3 \\ -8 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ -5 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -10 \\ 10 \end{pmatrix}$$

$$\vec{AQ} = \frac{3}{5} \vec{AB}$$

$$= \frac{3}{5} \begin{pmatrix} 5 \\ -10 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$$

$$\Rightarrow \vec{AQ} = \vec{Q} - \vec{A}$$

$$\vec{Q} = \vec{AQ} + \vec{A}$$

$$= \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$$

$$\Rightarrow \vec{CQ} = \vec{Q} - \vec{C} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Jawaban : E

12  $A(2, 3, -1) ; B(8, -3, 11)$

$$\Rightarrow \vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 8 \\ -3 \\ 11 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix}$$

$$\Rightarrow PB : AP = 5 : 1$$

$$\vec{AP} = \frac{1}{6} \vec{AB} = \frac{1}{6} \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{AP} = \vec{P} - \vec{A}$$

$$\vec{P} = \vec{AP} + \vec{A} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$P = 3i + 2j + k$$

Jawaban : A

13  $P(1, -2, -2) ; Q(1, -2, 6) ; RQ : PR = 3 : 5$

$$R = \frac{5 \begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}}{8}$$

$$= \frac{\begin{pmatrix} 5 \\ -10 \\ 30 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \\ -6 \end{pmatrix}}{8} = \frac{\begin{pmatrix} 8 \\ -16 \\ 24 \end{pmatrix}}{8} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$\Rightarrow R = (1, -2, 3)$$

$$\vec{PR} = \vec{R} - \vec{P} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$$

$$\vec{PQ} = \vec{Q} - \vec{P} = \begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$$

$$PR \cdot PQ = 0 + 0 + (5)(8) = 40$$

Jawaban : E

14  $A(2, -4, 3) ; B(12, -9, -17)$

$$AP : PB = 1 : 4$$

$$P = \frac{1 \begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}}{5}$$

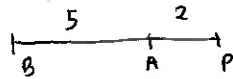
$$= \frac{\begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} + \begin{pmatrix} 8 \\ -16 \\ 12 \end{pmatrix}}{5} = \frac{\begin{pmatrix} 20 \\ -25 \\ -5 \end{pmatrix}}{5} = \begin{pmatrix} 4 \\ -5 \\ -1 \end{pmatrix}$$

$$|\vec{P}| = \sqrt{4^2 + (-5)^2 + (-1)^2} \\ = \sqrt{16 + 25 + 1} = \sqrt{42}$$

Jawaban : B

15)  $A(2, -4, 3)$  ;  $B(12, -9, -17)$

$AP : AB = 2 : 5$



$$\Rightarrow \vec{BA} = \vec{A} - \vec{B} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \\ 20 \end{pmatrix}$$

$$\Rightarrow \vec{AP} = \vec{P} - \vec{A} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} x-2 \\ y+4 \\ z-3 \end{pmatrix}$$

$\Rightarrow \frac{AP}{BA} = \frac{2}{5}$

$$\begin{aligned} \Rightarrow 5x-10 &= -20 \\ 5x &= -10 \\ x &= -2 \end{aligned}$$

$5AP = 2BA$

$$5 \begin{pmatrix} x-2 \\ y+4 \\ z-3 \end{pmatrix} = 2 \begin{pmatrix} -10 \\ 5 \\ 20 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow 5y+20 &= 10 \\ 5y &= -10 \\ y &= -2 \end{aligned}$$

$$\begin{pmatrix} 5x-10 \\ 5y+20 \\ 5z-15 \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow 5z-15 &= 40 \\ 5z &= 55 \\ z &= 11 \end{aligned}$$

$\Rightarrow \vec{P} = (-2, -2, 11)$

$$\begin{aligned} |\vec{P}| &= \sqrt{(-2)^2 + (-2)^2 + 11^2} \\ &= \sqrt{4+4+121} \\ &= \sqrt{129} = 11,3 \end{aligned}$$

Jawaban: C

16)  $A(1, -2, -8)$  ;  $B(3, -4, 0)$

$A\vec{P} = -3\vec{PB}$

$\frac{A\vec{P}}{PB} = \frac{-3}{1}$

$$P = \frac{-3 \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{-2} = \frac{\begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{-2}$$

$$= \frac{\begin{pmatrix} -8 \\ 10 \\ -8 \end{pmatrix}}{-2} = \begin{pmatrix} 4 \\ -5 \\ 4 \end{pmatrix}$$

$P = 4i - 5j + 4k$

Jawaban: D

17)  $\vec{a} = (3, 4, k_1)$  dan  $\vec{b} = (6, 1, k_2)$

$|\vec{a}-\vec{b}| = \sqrt{34}$  ,  $k_1, k_2 = 3$

$$\vec{a}-\vec{b} = \begin{pmatrix} 3 \\ 4 \\ k_1 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \\ k_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ k_1-k_2 \end{pmatrix}$$

$$|\vec{a}-\vec{b}| = \sqrt{(-3)^2 + 3^2 + (k_1-k_2)^2}$$

$$\sqrt{34} = \sqrt{9+9+k_1^2-2k_1k_2+k_2^2}$$

$$34 = 18 + k_1^2 - 2(3) + k_2^2$$

$$16 = k_1^2 + k_2^2 - 6$$

$$22 = k_1^2 + k_2^2$$

Jawaban: C

18)  $\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 6 \end{pmatrix}$

$$\vec{AC} = \vec{C} - \vec{A} = \begin{pmatrix} 6 \\ m \\ -6 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ m-5 \\ -6 \end{pmatrix}$$

$|\vec{AB}| = |\vec{AC}|$

$$\sqrt{25+1+36} = \sqrt{1+(m-5)^2+36}$$

$$25 = m^2 - 10m + 25$$

$$0 = m^2 - 10m$$

$$0 = m(m-10)$$

$m=0$  atau  $m=10$

Jawaban: D

$$9) \vec{OA} = i + j + 2k ; \vec{OB} = i + 2j + 3k$$

$$|\vec{PA}| = |\vec{OB}|$$

$$\Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$$

$$= (1, 2, 3) - (1, 1, 2) = (0, 1, 1)$$

$$\Rightarrow \vec{AP} = k \cdot \vec{AB}$$

$$= k (0, 1, 1)$$

$$= (0, k, k)$$

$$\Rightarrow |\vec{AP}| = |\vec{OB}|$$

$$\sqrt{k^2 + k^2} = \sqrt{1^2 + 2^2 + 3^2}$$

$$2k^2 = 1 + 4 + 9$$

$$2k^2 = 14$$

$$k^2 = 7$$

$$k = \pm\sqrt{7}$$

$$\Rightarrow \vec{OA} \cdot \vec{AP}$$

$$= (1, 1, 2) \cdot \sqrt{7} \cdot (0, 1, 1)$$

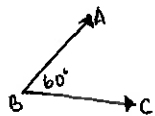
$$= \sqrt{7} (0 + 1 + 2)$$

$$= 3\sqrt{7}$$

Jawaban: C

$$10) \vec{OA} = i + k ; \vec{OB} = j + k ; \vec{OC} = mj + 4k$$

$$\angle ABC = 60^\circ$$



$$\Rightarrow \vec{BA} = \vec{OA} - \vec{OB}$$

$$= (1, 0, 1) - (0, 1, 1)$$

$$= (1, -1, 0)$$

$$\Rightarrow \vec{BC} = \vec{OC} - \vec{OB}$$

$$= (0, m, 4) - (0, 1, 1)$$

$$= (0, m-1, 3)$$

$$\Rightarrow |\vec{BA}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$|\vec{BC}| = \sqrt{(m-1)^2 + 3^2} = \sqrt{m^2 - 2m + 10}$$

$$\Rightarrow \vec{BA} \cdot \vec{BC} = |\vec{BA}| |\vec{BC}| \cos 60^\circ$$

$$(1, -1, 0) \cdot (0, m-1, 3) = \sqrt{2} (\sqrt{m^2 - 2m + 10}) \cdot \frac{1}{2}$$

$$(-m+1) = \left( \sqrt{2} \sqrt{m^2 - 2m + 10} \cdot \frac{1}{2} \right)^2$$

$$1 - 2m + m^2 = \frac{1}{2} \cdot m^2 - 2m + 10$$

$$2 - 4m + 2m^2 = m^2 - 2m + 10$$

$$\Rightarrow m^2 - 2m - 8 = 0$$

$$(m-4)(m+2) = 0$$

$$m_1 = 4 \quad m_2 = -2$$

$$\Rightarrow (m_1 + m_2)^2 = (4 - 2)^2 = 4$$

Jawaban: B

$$11) \vec{a} = 2i - 3j + 5k, \vec{b} = -3i - 5j + 2k$$

$$\Rightarrow \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$= \frac{(-6 + 15 + 10)}{\sqrt{4+9+25} \cdot \sqrt{9+25+4}} = \frac{19}{\sqrt{38} \cdot \sqrt{38}} = \frac{19}{38} = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\Rightarrow \tan 60^\circ = \sqrt{3}$$

Jawaban: E

$$12) |\vec{p}| = 10 ; |\vec{q}| = 6 ; \theta = 60^\circ$$

$$P \cdot q = |\vec{p}| |\vec{q}| \cos \theta$$

$$= 10 \cdot 6 \cdot \cos 60^\circ$$

$$= 60 \cdot \frac{1}{2} = 30$$

$$\Rightarrow |\vec{p} - \vec{q}|^2 = |\vec{p}|^2 - 2 \vec{p} \cdot \vec{q} + |\vec{q}|^2$$

$$= 10^2 - 2 \cdot 30 + 6^2$$

$$= 100 - 60 + 36$$

$$= 76$$

$$|\vec{p} - \vec{q}| = \sqrt{76} = 2\sqrt{19}$$

Jawaban: B



$$\begin{aligned} \boxed{13} \quad g &= AB = B - A = (4, 1, -1) - (2, 4, -2) \\ &= (2, -3, 1) \end{aligned}$$

$$\begin{aligned} h &= PQ = Q - P = (8, 2, 1) - (7, 0, 2) \\ &= (1, 2, -3) \end{aligned}$$

$$|g| = \sqrt{4+9+1} = \sqrt{14}$$

$$|h| = \sqrt{1+4+9} = \sqrt{14}$$

$$\Rightarrow \cos \theta = \frac{g \cdot h}{|g| \cdot |h|} = \frac{(2-6-3)}{\sqrt{14} \cdot \sqrt{14}} = \frac{-7}{14} = -\frac{1}{2}$$

$$\theta = 120^\circ$$

Jawaban: E

$$\boxed{14} \quad \bar{A} (2k, k+1); \bar{B} (4k, k+5), \bar{C} (10+k, 0)$$

$$T = \frac{\bar{A} + \bar{B}}{2} = \frac{(6k, 2k+6)}{2} = (3k, k+3)$$

$$\begin{aligned} \bar{CT} &= \bar{T} - \bar{C} = (3k, k+3) - (10+k, 0) \\ &= (2k-10, k+3) \end{aligned}$$

$$|CT| = |OT|$$

$$\sqrt{(2k-10)^2 + (k+3)^2} = \sqrt{(3k)^2 + (k+3)^2}$$

$$4k^2 - 40k + 100 + k^2 + 6k + 9 = 9k^2 + k^2 + 6k + 9$$

$$0 = 5k^2 + 40k - 100$$

$$(5k-10)(k+10) = 0$$

$$\begin{aligned} k &= \frac{10}{5} & k &= -10 \\ &= 2 \end{aligned}$$

$\therefore$  nilai  $k$  positif adalah 2

Jawaban: B

$$\boxed{15} \quad A = (-7, 1, -5)$$

$$B = (2, 2, -2)$$

$$\Rightarrow |A| = \sqrt{49+1+25} = \sqrt{75} = 5\sqrt{3}$$

$$|B| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

$$\Rightarrow |A| - |B| = 5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$$

Jawaban: C

$$\boxed{16} \quad A(6, -2, -6); B(3, 4, 6); C(9, x, y)$$

$$\Rightarrow \bar{AB} = n \cdot \bar{AC}$$

$$\bar{B} - \bar{A} = n(\bar{C} - \bar{A})$$

$$\begin{pmatrix} -3 \\ 6 \\ 12 \end{pmatrix} = n \begin{pmatrix} 3 \\ x+2 \\ y+6 \end{pmatrix}$$

$$\Rightarrow -3 = 3n$$

$$-1 = n$$

$$\Rightarrow (x+2)(-1) = 6$$

$$x+2 = -6$$

$$x = -8$$

$$\Rightarrow (y+6)(-1) = 12$$

$$y+6 = -12$$

$$y = -18$$

$$\Rightarrow x - y = -8 - (-18) = 10$$

Jawaban: D

$$\boxed{17} \quad \bar{a} = (x, 4, 7); \bar{b} = (6, y, 14)$$

$$\bar{a} = n \cdot \bar{b}$$

$$\Rightarrow 7 = n \cdot 14$$

$$\frac{1}{2} = n$$

$$\begin{pmatrix} x \\ 4 \\ 7 \end{pmatrix} = n \begin{pmatrix} 6 \\ y \\ 14 \end{pmatrix}$$

$$\Rightarrow x = 6n$$

$$= 6\left(\frac{1}{2}\right)$$

$$= 3$$

$$\Rightarrow yn = 4$$

$$y = \frac{4}{n} = \frac{4}{\frac{1}{2}} = 8$$

$$\Rightarrow (x-y)^2 = (3-8)^2 = 25$$

Jawaban: C

$$18] \quad a = (-1, 2, 2) ; b = (3, -5, 2)$$

$$c = (0, 1, 8)$$

$$\Rightarrow C = pa + qb$$

$$\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} = \begin{pmatrix} -p \\ 2p \\ 2p \end{pmatrix} + \begin{pmatrix} 3q \\ -5q \\ 2q \end{pmatrix}$$

$$\begin{aligned} \Rightarrow 2p - 5q &= 1 \\ 2p + 2q &= 8 \\ \hline -7q &= -7 \\ q &= 1 \\ p &= 3 \end{aligned} \quad \Rightarrow (p+q)^2 = (3+1)^2 = 4^2 = 16$$

Jawaban : D

$$19] \quad \vec{OA} = (2, -2, 4) ; \vec{OB} = (-2, 4, -2)$$

$$\begin{aligned} \Rightarrow \vec{OP} &= 2\vec{OA} & \Rightarrow \vec{OQ} &= -3\vec{OB} \\ &= 2(2, -2, 4) & &= -3(-2, 4, -2) \\ &= (4, -4, 8) & &= (6, -12, 6) \end{aligned}$$

$$\begin{aligned} \Rightarrow \vec{PQ} &= \vec{OQ} - \vec{OP} \\ &= (6, -12, 6) - (4, -4, 8) \\ &= (2, -8, -2) \end{aligned}$$

$$\vec{PQ} = 2i - 8j - 2k$$

Jawaban : D

$$20] \quad |\vec{a}| = 4 ; |\vec{b}| = 2 ; \vec{a} \cdot (\vec{a} - \vec{b}) = 12$$

$$\begin{aligned} \Rightarrow |\vec{a}|^2 - \vec{a} \cdot \vec{b} &= 12 \\ 16 - \vec{a} \cdot \vec{b} &= 12 \\ \vec{a} \cdot \vec{b} &= 4 \end{aligned}$$

$$\begin{aligned} \Rightarrow |\vec{a} + \vec{b}|^2 &= |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \\ &= 4^2 + 2(4) + 2^2 \\ &= 16 + 8 + 4 \\ &= 28 \\ |\vec{a} + \vec{b}| &= \sqrt{28} = 2\sqrt{7} \end{aligned}$$

Jawaban : B

$$21] \quad \vec{a} = (-1, 1, 2) ; \vec{b} = (x, -2, 3)$$

$$\Rightarrow (\vec{a} + \vec{b}) = (x-1, -1, 5)$$

$$\Rightarrow (\vec{b} - \vec{a}) = (x+1, -3, 1)$$

$$(\vec{a} + \vec{b}) \cdot (\vec{b} - \vec{a}) = 11$$

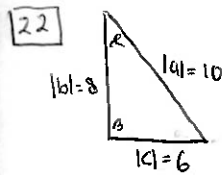
$$(x-1)(x+1) + (-1)(-3) + (5)(1) = 11$$

$$x^2 - 1 + 3 + 5 = 11$$

$$x^2 = 4$$

$$x = \pm 2$$

Jawaban : D



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\Rightarrow \cos \beta = \cos 90^\circ = 0$$

$$\begin{aligned} \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} &= |\vec{a}| |\vec{b}| \cos \alpha + |\vec{b}| |\vec{c}| \cos \beta \\ &= 8 \cdot 10 \left( \frac{8}{10} \right) + 8 \cdot 6 \cdot 0 \\ &= 64 \end{aligned}$$

Jawaban : A

$$23] \quad \vec{p} \cdot \vec{p} = |\vec{p}|^2 = 3 ; |\vec{q}| = 4$$

$$\begin{aligned} \Rightarrow |\vec{p} + \vec{q}|^2 &= |\vec{p}|^2 + |\vec{q}|^2 + 2|\vec{p}| |\vec{q}| \cos 30^\circ \\ &= 3 + 16 + 2(\sqrt{3})(4) \frac{1}{2} \sqrt{3} \\ &= 19 + 12 \\ &= 31 \\ |\vec{p} + \vec{q}| &= \sqrt{31} \end{aligned}$$

$$\begin{aligned} \Rightarrow |\vec{p} - \vec{q}|^2 &= |\vec{p}|^2 + |\vec{q}|^2 - 2|\vec{p}| |\vec{q}| \cos 30^\circ \\ &= 3 + 16 - 2(\sqrt{3})(4) \frac{1}{2} \sqrt{3} \\ &= 19 - 12 \\ &= 7 \end{aligned}$$

$$|\vec{p} - \vec{q}| = \sqrt{7}$$

$$\begin{aligned} \Rightarrow |\vec{p} + \vec{q}| |\vec{p} - \vec{q}| &= \sqrt{31} \cdot \sqrt{7} \\ &= \sqrt{217} \end{aligned}$$

Jawaban : E

24)  $BM : MC = 1 : 3$  ;  $\overrightarrow{BM} = \vec{u}$  ;  $\overrightarrow{CN} = \vec{v}$

$MD : DA = 1 : 4$   $\overrightarrow{BD} = \frac{1}{n} \overrightarrow{BN}$

$\Rightarrow \frac{BM}{MC} = \frac{1}{3}$

$MC = 3BM$

$\Rightarrow \frac{MD}{DA} = \frac{1}{4}$

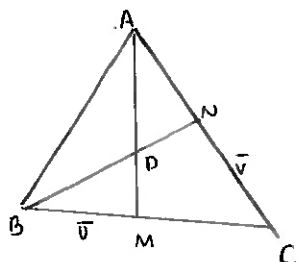
$DA = 4MD$

$\Rightarrow MA = MD + DA$   
 $= 5MD$

$\Rightarrow MA = MC + CA$

$5MD = 3\vec{u} + 2\vec{v}$

$MD = \frac{1}{5} (3\vec{u} + 2\vec{v})$



$\Rightarrow \overrightarrow{BD} = \overrightarrow{BM} + \overrightarrow{MD}$   
 $= \vec{u} + \left( \frac{3}{5}\vec{u} + \frac{2}{5}\vec{v} \right)$   
 $= \frac{8}{5}\vec{u} + \frac{2}{5}\vec{v}$

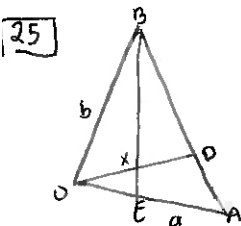
$\Rightarrow \overrightarrow{BN} = \overrightarrow{BC} + \overrightarrow{CN}$   
 $= 4\vec{u} + \vec{v}$

$\Rightarrow \overrightarrow{BD} = \frac{1}{n} \overrightarrow{BN}$

$\frac{8}{5}\vec{u} + \frac{2}{5}\vec{v} = \frac{1}{n} (4\vec{u} + \vec{v})$

$\Rightarrow \frac{1}{n} = \frac{2}{5} \Rightarrow n = \frac{5}{2}$

Jawaban : E



$\Rightarrow OA = a$

$OB = b$

$AD = \frac{1}{3} AB = \frac{1}{3} (b-a)$

$OE = \frac{1}{3} OA = \frac{1}{3} a$

$\Rightarrow \overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$   
 $= b - \frac{2}{3}(b-a)$   
 $= \frac{2}{3}a + \frac{1}{3}b$

$\Rightarrow \overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$   
 $= -b + \frac{1}{3}a$

$\Rightarrow \overrightarrow{OB} = 0x + xB$

$b = k \cdot \overrightarrow{OD} - l \cdot \overrightarrow{BE}$

$b = k \left( \frac{2}{3}a + \frac{1}{3}b \right) - l \left( -b + \frac{1}{3}a \right)$

$b = \left( \frac{2k}{3} - \frac{l}{3} \right) a + \left( \frac{k}{3} + l \right) b$

$\Rightarrow \frac{2k}{3} - \frac{l}{3} = 0$

$\Rightarrow \frac{k}{3} + l = 1$

$2k - l = 0 \dots (i)$

$k + 3l = 3 \dots (ii)$

dari (i) & (ii)

$\Rightarrow \begin{matrix} 2k - l = 0 & \times 1 \\ k + 3l = 3 & \times 2 \end{matrix}$

$2k - l = 0$

$2k + 6l = 6$

$-7l = -6$

$l = \frac{6}{7}$

$\Rightarrow k = 3 - 3\left(\frac{6}{7}\right)$   
 $= \frac{3}{7}$

$\therefore k = \frac{3}{7}$  dan  $l = \frac{6}{7}$

Jawaban : A

26)  $\overrightarrow{GM} = 2\vec{u}$  ;  $\overrightarrow{GN} = 2\vec{v}$  ;  $\overrightarrow{GK} = \frac{5}{2} \overrightarrow{LK}$  ;  $\overrightarrow{MN} = k \cdot \overrightarrow{LK}$

$\Rightarrow \overrightarrow{MN} = \overrightarrow{MG} + \overrightarrow{GN}$   
 $= 2\vec{u} + 2\vec{v}$

$\Rightarrow \overrightarrow{HK} = \overrightarrow{HG} + \overrightarrow{GK}$   
 $= 4\vec{u} + \frac{5}{2} \overrightarrow{LK}$

$\Rightarrow \overrightarrow{HN} = \overrightarrow{HM} + \overrightarrow{MN}$   
 $= 2\vec{u} + 2\vec{u} + 2\vec{v}$   
 $= 4\vec{u} + 2\vec{v}$

$\Rightarrow \overrightarrow{HK} = \overrightarrow{HL} + \overrightarrow{LK}$   
 $4\vec{u} + \frac{5}{2} \overrightarrow{LK} = 8\vec{u} + 4\vec{v} + \overrightarrow{LK}$

$\frac{3}{2} \overrightarrow{LK} = 4\vec{u} + 4\vec{v}$

$\overrightarrow{LK} = \frac{8\vec{u} + 8\vec{v}}{3}$

$\Rightarrow \overrightarrow{MN} = k \cdot \overrightarrow{LK}$

$2\vec{u} + 2\vec{v} = k \left( \frac{8}{3}\vec{u} + \frac{8}{3}\vec{v} \right)$

$\Rightarrow 2 = \frac{8k}{3}$

$\frac{6}{8} = k$

$\frac{3}{4} = k$

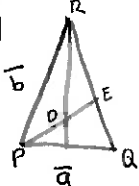
$\frac{3}{4}$

Jawaban : B

27  $\vec{v} = \vec{a}$  \*  $\vec{v} = \frac{\vec{q} + \vec{s}}{2}$   
 $\vec{v} - \vec{u} = \vec{a}$   $\vec{u} = \frac{\vec{p} + \vec{r}}{2}$

$$\begin{aligned} \Rightarrow \vec{PQ} + \vec{PS} + \vec{RQ} + \vec{RS} \\ = \vec{Q} - \vec{P} + \vec{S} - \vec{P} + \vec{Q} - \vec{R} + \vec{S} - \vec{R} \\ = \vec{Q} - \vec{P} + \vec{S} - \vec{P} + \vec{Q} - \vec{R} + \vec{S} - \vec{R} \\ = 2\vec{Q} - 2\vec{P} + 2\vec{S} - 2\vec{R} \\ = 2(\vec{Q} + \vec{S}) - 2(\vec{P} + \vec{R}) \\ = 2 \cdot 2\vec{v} - 2 \cdot 2\vec{u} \\ = 4\vec{v} - 4\vec{u} \\ = 4(\vec{v} - \vec{u}) = 4\vec{a} \end{aligned}$$

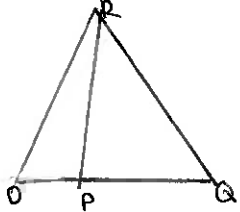
Jawaban: E

28   $\Rightarrow \vec{RQ} = \vec{b} - \vec{a}$   
 $\Rightarrow \vec{DE} = \frac{1}{3} \vec{PE}$

$$\begin{aligned} \Rightarrow \vec{PE} &= \vec{PQ} + \vec{QE} \\ &= \vec{a} + \frac{1}{2} \vec{QR} \\ &= \vec{a} + \frac{1}{2} (\vec{b} - \vec{a}) \\ &= \frac{1}{2} \vec{a} + \frac{1}{2} \vec{b} \end{aligned}$$

$$\Rightarrow \vec{DE} = \frac{1}{3} \vec{PE} = \frac{1}{3} \left( \frac{1}{2} \vec{a} + \frac{1}{2} \vec{b} \right) = \frac{\vec{a}}{6} + \frac{\vec{b}}{6}$$

Jawaban: A

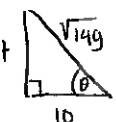
29   $\Rightarrow \cos \angle ROP = \frac{\vec{OR} \cdot \vec{OP}}{|\vec{OR}| |\vec{OP}|}$   
 $\Rightarrow \vec{OR} = \vec{OP} + \vec{PR}$   
 $= 12\vec{a} + 8\vec{a} + 14\vec{b}$   
 $= 20\vec{a} + 14\vec{b}$

$$\Rightarrow |\vec{OR}| = \sqrt{400 + 196} = \sqrt{596} = 2\sqrt{149}$$

$$|\vec{OP}| = \sqrt{144} = 12$$

$$\Rightarrow \cos \angle ROP = \frac{240}{2\sqrt{149} \cdot 12} = \frac{10}{\sqrt{149}}$$

$$\Rightarrow \tan \angle ROP = \frac{7}{10}$$



$$\Rightarrow \cos \angle RQP = \frac{\vec{QR} \cdot \vec{QP}}{|\vec{QR}| |\vec{QP}|} \Rightarrow \vec{OP} = 12\vec{a}$$

$$\vec{PR} = 8\vec{a} + 14\vec{b}$$

$$\vec{OP} = \frac{3}{8} \vec{OQ}$$

$$\vec{OQ} = 32\vec{a}$$

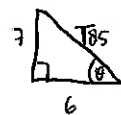
$$\Rightarrow \vec{QP} = -20\vec{a}$$

$$|\vec{QP}| = 20$$

$$\begin{aligned} \Rightarrow \vec{QR} &= \vec{QP} + \vec{PR} \\ &= -20\vec{a} + 8\vec{a} + 14\vec{b} \\ &= -12\vec{a} + 14\vec{b} \end{aligned}$$

$$|\vec{QR}| = \sqrt{144 + 196} = \sqrt{340} = 2\sqrt{85}$$

$$\Rightarrow \cos \angle RQP = \frac{(240)}{2\sqrt{85} \cdot 20} = \frac{6}{\sqrt{85}}$$



$$\Rightarrow \tan \theta = \frac{7}{6}$$

$$\therefore \tan \angle ROP = \frac{7}{10} \text{ \& \; } \tan \angle RQP = \frac{7}{6}$$

Jawaban: E

30  $\vec{OA} = 4\vec{a}$   $\Rightarrow \vec{OP} : \vec{PC} = 3 : 1$   
 $\vec{OC} = 8\vec{c}$   $\vec{OP} = \frac{3}{4} \vec{OC} = 6\vec{c}$

$$\Rightarrow \vec{AB} = \vec{OC} \quad \vec{PC} = \frac{1}{4} \vec{OC} = 2\vec{c}$$

$$\Rightarrow L_{\triangle ABQ} = \frac{1}{2} |\vec{BQ}| \cdot |\vec{AB}|$$

$$64 = \frac{1}{2} |\vec{BQ}| \cdot 8$$

$$16 = |\vec{BQ}|$$

$$\Rightarrow \frac{PC}{AB} = \frac{QC}{QB} \Rightarrow |\vec{BQ}| = \left( \frac{4}{3} + 4 \right) |\vec{a}|$$

$$\frac{2}{8} = \frac{x}{x+4}$$

$$16 = \frac{16}{3} |\vec{a}|$$

$$2x+8 = 8x$$

$$3 = |\vec{a}|$$

$$x = \frac{8}{6} = \frac{4}{3} \vec{a}$$

Jawaban: A

## B. Uraian

1) waktu  $t \rightarrow (t, t^2 - 2t, 0)$

a)  $t=1 \rightarrow \vec{P} = (1, -1, 0)$

$\Rightarrow$  vektor posisi  $\vec{P} = \vec{i} - \vec{j}$

$t=2 \rightarrow \vec{Q} = (2, 0, 0)$

$\Rightarrow$  vektor posisi  $\vec{Q} = 2\vec{i}$

b)  $|\vec{P}| = \sqrt{1+1} = \sqrt{2}$

$|\vec{Q}| = \sqrt{2^2} = 2$

$\Rightarrow \cos \theta = \frac{\vec{P} \cdot \vec{Q}}{|\vec{P}| |\vec{Q}|} = \frac{2}{2 \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45^\circ$

2)  $A = (-1, 3, 1)$   $C = (0, 2, 5)$

$B = (1, 6, 7)$   $D = (1, 4, 10)$

a)  $\vec{AB} = \vec{B} - \vec{A} = (1, 6, 7) - (-1, 3, 1)$   
 $= (2, 3, 6)$

$\vec{CD} = \vec{D} - \vec{C} = (1, 4, 10) - (0, 2, 5)$   
 $= (1, 2, 5)$

b)  $\vec{U} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$  tegak lurus pada  $\vec{AB}$  dan  $\vec{CD}$

$\Rightarrow \vec{U} \cdot \vec{AB} = 0 \quad \Rightarrow \vec{U} \cdot \vec{CD} = 0$

$2x + 3y + 6 = 0$

$x + 2y + 5 = 0$

$2x + 3y = -6 \dots (i)$

$x + 2y = -5 \dots (ii)$

dari (i) dan (ii)

$$\begin{array}{rcl} 2x + 3y = -6 & | \times 1 & \\ x + 2y = -5 & | \times 2 & \end{array}$$

$$\begin{array}{rcl} 2x + 3y = -6 & & \\ 2x + 4y = -10 & - & \\ \hline -y = 4 & & \end{array}$$

$\Rightarrow x - 8 = -5$   
 $x = 3$

$y = -4$

$\therefore x = 3, y = -4, \vec{U} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

3)  $P = (2, 4, 6); Q = (6, 6, y); R = (14, 2x, -6)$

$\Rightarrow \vec{PQ} = \vec{Q} - \vec{P} = (6, 6, y) - (2, 4, 6)$   
 $= (4, 2, y-6)$

$\Rightarrow \vec{QR} = \vec{R} - \vec{Q} = (14, 2x, -6) - (6, 6, y)$   
 $= (8, 2x-6, -6-y)$

$\Rightarrow P, Q, R$  segaris

$\vec{PQ} = n \cdot \vec{QR} \quad \Rightarrow 4 = n \cdot 8$

$\begin{pmatrix} 4 \\ 2 \\ y-6 \end{pmatrix} = n \cdot \begin{pmatrix} 8 \\ 2x-6 \\ -6-y \end{pmatrix}$

$\frac{1}{2} = n$

$\Rightarrow 2 = \frac{1}{2}(2x-6) \quad \Rightarrow y-6 = \frac{1}{2}(-6-y)$

$4 = 2x-6$

$2y-12 = -6-y$

$10 = 2x$

$3y = 6$

$\boxed{5 = x}$

$\boxed{y = 2}$

$\Rightarrow \vec{PQ} = n \cdot \vec{QR}$

$\therefore \vec{PQ} : \vec{QR} = 1 : 2$

$\vec{PQ} = \frac{1}{2} \vec{QR}$

$\frac{\vec{PQ}}{\vec{QR}} = \frac{1}{2}$

4)  $\vec{U} = 2\vec{i} - \vec{j} + 2\vec{k}$  dan  $\vec{V} = 4\vec{i} + 10\vec{j} - 8\vec{k}$

$(\vec{U} + n\vec{V}) \perp \vec{U}$

$\Rightarrow n\vec{V} = (4n, 10n, -8n)$

$\vec{U} + n\vec{V} = (2, -1, 2) + (4n, 10n, -8n)$   
 $= (2+4n, -1+10n, 2-8n)$

$\Rightarrow (\vec{U} + n\vec{V}) \cdot \vec{U} = 0$

$4+8n + 1-10n + 4-16n = 0$

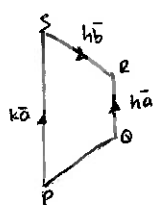
$9 - 18n = 0$

$18n = 9$

$n = \frac{9}{18}$

$= \frac{1}{2}$

5



$$\Rightarrow \vec{PQ} = 8\vec{a} + \left(\frac{k-b}{2}\right)\vec{b}$$

$$\Rightarrow \vec{PQ} = \vec{PS} + \vec{SR} + \vec{RQ}$$

$$8\vec{a} + \left(\frac{k-b}{2}\right)\vec{b} = k\vec{a} + h\vec{b} - h\vec{a}$$

$$8\vec{a} + \left(\frac{k-b}{2}\right)\vec{b} = (k-h)\vec{a} + h\vec{b}$$

$$\Rightarrow 8 = k-h \dots (i)$$

$$\Rightarrow \frac{k-b}{2} = h$$

$$k-b = 2h$$

$$k-2h = b \dots (ii)$$

$$\Rightarrow \begin{array}{l} k-h=8 \\ k-2h=b \\ \hline h=2 \\ k=10 \end{array}$$

a) nilai  $h=2$ , nilai  $k=10$

$$b) L \Delta QRS = \frac{1}{2} QR \cdot t$$

$$40 = \frac{1}{2} \cdot 2\vec{a} \cdot t$$

$$40 = \vec{a} \cdot t$$

$$L \Delta PQS = \frac{1}{2} PS \cdot t$$

$$= \frac{1}{2} 10\vec{a} \cdot t$$

$$= 5 \times 40$$

$$= 80 \text{ satuan luas}$$

6) misal panjang proyeksi  $\vec{a}$  pada  $\vec{b} = \vec{c}$

$$\vec{c} = 3|\vec{b}|$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 3|\vec{b}|$$

$$\frac{|\vec{a}||\vec{b}| \cos \alpha}{|\vec{b}|} = 3|\vec{b}|$$

$$|\vec{a}| \cos \alpha = 3|\vec{b}|$$

$$\frac{|\vec{a}|}{|\vec{b}|} = \frac{3}{\cos \alpha}$$

$$a) |\vec{a}| : |\vec{b}| = 3 : \cos \alpha$$

$$b) \alpha = 60^\circ ; |\vec{a}| = |\vec{b}|$$

$$\Rightarrow \frac{|\vec{a}|}{|\vec{b}|} = \frac{3}{\cos \alpha}$$

$$= \frac{3}{\cos 60} = \frac{3}{\frac{1}{2}} = 6$$

$$\Rightarrow |\vec{a}| = 6|\vec{b}|$$

$$6 = 6|\vec{b}|$$

$$1 = |\vec{b}|$$

$$\begin{aligned} \Rightarrow |\vec{a} + \vec{b}| &= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos 60} \\ &= \sqrt{36 + 1 + 2(1)(6) \cdot \frac{1}{2}} \\ &= \sqrt{37 + 6} \\ &= \sqrt{43} \end{aligned}$$

$$7) \vec{a} = (x, y), \vec{b} = (3, 4), |\vec{a}| = \sqrt{5}$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 2$$

$$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{5}$$

$$x^2 + y^2 = 5$$

$$\frac{3x + 4y}{5} = 2$$

$$y = \sqrt{5 - x^2}$$

$$3x + 4y = 10$$

$$\Rightarrow 3x + 4(\sqrt{5 - x^2}) = 10$$

$$4(\sqrt{5 - x^2}) = 10 - 3x$$

$$\Rightarrow 16(5 - x^2) = 100 - 60x + 9x^2$$

$$80 - 16x^2 = 100 - 60x + 9x^2$$

$$25x^2 - 60x + 20 = 0$$

$$5x^2 - 12x + 4 = 0$$

$$(5x - 2)(x - 2) = 0$$

$$x_1 = \frac{2}{5} \quad x_2 = 2$$

$$\Rightarrow x_1 = \frac{2}{5} \longrightarrow y_1 = \frac{11}{5}$$

$$x_2 = 2 \longrightarrow y_2 = 1$$

$$\therefore \vec{a} = \frac{2}{5}\vec{i} + \frac{11}{5}\vec{j} \text{ dan } \vec{a} = 2\vec{i} + \vec{j}$$

$$b.) \vec{a} = 2\vec{i} + 6\vec{j} + 2\vec{k}$$

$$b = -3\vec{i} + x\vec{k}$$

$$\text{proyeksi } \vec{a} \text{ thdp } \vec{b} = |\vec{p}| = \frac{2}{5}$$

$$\Rightarrow |\vec{p}| = \frac{a \cdot b}{|b|}$$

$$\frac{2}{5} = \frac{-6 + 2x}{\sqrt{9+x^2}}$$

$$2\sqrt{9+x^2} = -30 + 10x$$

$$4(9+x^2) = (-30+10x)^2$$

$$36 + 4x^2 = 900 + 600x + 100x^2$$

$$96x^2 + 600x + 864 = 0$$

$$4x^2 - 25x + 36 = 0$$

$$(4x-9)(x-4) = 0$$

$$x_1 = \frac{9}{4} \quad x_2 = 4$$

$$[8] a.) \vec{OP} = \vec{P}$$

$$OA:AP = 2:1$$

$$\Rightarrow \vec{OA} = \frac{2}{3} \vec{OP}$$

$$= \frac{2}{3} \vec{P}$$

$$\Rightarrow \vec{QA} = \vec{QO} + \vec{OA}$$

$$= -\vec{q} + \frac{2}{3} \vec{P}$$

$$= \frac{2}{3} \vec{P} - \vec{q}$$

$$b.) \vec{AR} : \vec{RQ} = 1:k$$

$$\Rightarrow \vec{RQ} = \frac{k}{k+1} \cdot \vec{QA}$$

$$= \frac{k}{k+1} \left( \frac{2}{3} \vec{P} - \vec{q} \right)$$

$$= \frac{2k}{3k+3} \vec{P} - \frac{k}{k+1} \vec{q}$$

$$\Rightarrow -\frac{2}{9} (2\vec{P} - 3\vec{q}) = \frac{2k}{3k+3} \vec{P} - \frac{k}{k+1} \vec{q}$$

$$\Rightarrow \frac{-4}{9} = \frac{2k}{3k+3}$$

$$-12k + 12 = 18k$$

$$-12 = 30k$$

$$\frac{-12}{30} = k$$

$$\frac{-2}{5} = k$$

$$[9] \vec{I}_1 = (2, 9) ; \vec{I}_2 = (-8, 4) ; \vec{I}_3 = (x, y)$$

$$\vec{I}_1 \vec{I}_3 : \vec{I}_3 \vec{I}_2 = 3:2$$

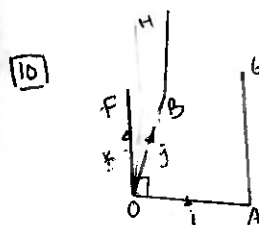
$$\vec{I}_3 = \frac{3\vec{I}_2 + 2\vec{I}_1}{5} = \frac{3\begin{pmatrix} -8 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 9 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} -24 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ 18 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} -20 \\ 30 \end{pmatrix}}{5} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

$$a) \text{posisi } \vec{I}_3 = (-4, 6)$$

$$b.) |\vec{I}_3| = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$



$$\Rightarrow OF = 10 \text{ m}$$

$$OA = 4 \text{ m}$$

$$AH = 14 \text{ m}$$

$$OB = 8 \text{ m}$$

$$BH = 18 \text{ m}$$

$$a.) \vec{OF} = 10\vec{k}$$

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$= 4\vec{i} + 14\vec{k}$$

$$\vec{OH} = \vec{OB} + \vec{BH}$$

$$= 8\vec{j} + 18\vec{k}$$

$$b.) \vec{AF} = \vec{AO} + \vec{OF}$$

$$= -4\vec{i} + 10\vec{k}$$

$$\text{titik tengah AF} = \frac{\vec{AF}}{2} = \frac{-4\vec{i} + 10\vec{k}}{2} = \vec{P}$$

$$\Rightarrow \vec{GH} = \vec{OH} - \vec{OG} = (0, 8, 18) - (4, 0, 14)$$

$$= (-4, 8, 4)$$

$$\Rightarrow \vec{P} \rightarrow \vec{GH} = \vec{GH} - \vec{P} = (-4, 8, 4) - (-2, 0, 5)$$

$$= (-2, 8, -1)$$

$$|\vec{P}| = \sqrt{4+64+1}$$

$$= \sqrt{69}$$

$\therefore$  Panjang garis adalah  $\sqrt{69} \text{ m}$

# Latihan Soal akhir Bab 7

11  $A(-2, 2, -5) ; B(3, -8, 5) ; C(-1, -3, 0)$

$\Rightarrow \vec{AB} = \vec{B} - \vec{A} \quad \Rightarrow \vec{AQ} : \vec{QB} = 3 : 2$

$$= \begin{pmatrix} 3 \\ -8 \\ 5 \end{pmatrix} - \begin{pmatrix} -2 \\ 2 \\ -5 \end{pmatrix}$$

$$\vec{AQ} = \frac{3}{5} \vec{AB}$$

$$= \begin{pmatrix} 5 \\ -10 \\ 10 \end{pmatrix}$$

$$= \frac{3}{5} \begin{pmatrix} 5 \\ -10 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix}$$

$\Rightarrow \vec{AQ} = \vec{Q} - \vec{A}$

$$\vec{Q} = \vec{AQ} + \vec{A}$$

$$= \begin{pmatrix} 3 \\ -6 \\ 6 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$$

$\Rightarrow \vec{CQ} = \vec{Q} - \vec{C} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Jawaban : E

12  $A(2, 3, -1) ; B(8, -3, 11)$

$\Rightarrow \vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 8 \\ -3 \\ 11 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} -6 \\ -6 \\ 12 \end{pmatrix}$

$\Rightarrow \vec{PB} : \vec{AP} = 5 : 1$

$$\vec{AP} = \frac{1}{6} \vec{AB} = \frac{1}{6} \begin{pmatrix} -6 \\ -6 \\ 12 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

$$\vec{AP} = \vec{P} - \vec{A}$$

$$\vec{P} = \vec{AP} + \vec{A} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$P = 3i + 2j + k$$

Jawaban : A

13  $P(1, -2, -2) ; Q(1, -2, 6) ; RQ : PR = 3 : 5$

$$R = \frac{5 \begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}}{8}$$

$$= \frac{\begin{pmatrix} 5 \\ -10 \\ 30 \end{pmatrix} + \begin{pmatrix} 3 \\ -6 \\ -6 \end{pmatrix}}{8} = \frac{\begin{pmatrix} 8 \\ -16 \\ 24 \end{pmatrix}}{8} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$\Rightarrow R = (1, -2, 3)$

$$\vec{PR} = \vec{R} - \vec{P} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$$

$$\vec{PQ} = \vec{Q} - \vec{P} = \begin{pmatrix} 1 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$$

$$PR \cdot PQ = 0 + 0 + (5)(8) = 40$$

Jawaban : E

14  $A(2, -4, 3) ; B(12, -9, -17)$

$\vec{AP} : \vec{PB} = 1 : 4$

$$P = \frac{1 \begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} + \begin{pmatrix} 8 \\ -16 \\ 12 \end{pmatrix}}{5} = \frac{\begin{pmatrix} 20 \\ -25 \\ -5 \end{pmatrix}}{5} = \begin{pmatrix} 4 \\ -5 \\ -1 \end{pmatrix}$$

$$|\vec{P}| = \sqrt{4^2 + (-5)^2 + (-1)^2}$$

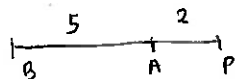
$$= \sqrt{16 + 25 + 1} = \sqrt{42}$$

Jawaban : B



5]  $A(2, -4, 3) ; B(12, -9, -17)$

$AP : AB = 2 : 5$



$$\Rightarrow \vec{BA} = \vec{A} - \vec{B} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 12 \\ -9 \\ -17 \end{pmatrix} = \begin{pmatrix} -10 \\ 5 \\ 20 \end{pmatrix}$$

$$\Rightarrow \vec{AP} = \vec{P} - \vec{A} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} = \begin{pmatrix} x-2 \\ y+4 \\ z-3 \end{pmatrix}$$

$$\Rightarrow \frac{AP}{BA} = \frac{2}{5}$$

$$\Rightarrow 5x-10 = -20 \\ 5x = -10 \\ x = -2$$

$$5AP = 2BA$$

$$5 \begin{pmatrix} x-2 \\ y+4 \\ z-3 \end{pmatrix} = 2 \begin{pmatrix} -10 \\ 5 \\ 20 \end{pmatrix}$$

$$\Rightarrow 5y+20 = 10 \\ 5y = -10 \\ y = -2$$

$$\begin{pmatrix} 5x-10 \\ 5y+20 \\ 5z-15 \end{pmatrix} = \begin{pmatrix} -20 \\ 10 \\ 40 \end{pmatrix}$$

$$\Rightarrow 5z-15 = 40 \\ 5z = 55 \\ z = 11$$

$$\Rightarrow \vec{P} = (-2, -2, 11)$$

$$|\vec{P}| = \sqrt{(-2)^2 + (-2)^2 + 11^2} \\ = \sqrt{4+4+121} \\ = \sqrt{129} = 11,3$$

Jawaban: C

6]  $A(1, -2, -8) ; B(3, -4, 0)$

$$\vec{AP} = -3\vec{PB}$$

$$\frac{AP}{PB} = \frac{-3}{1}$$

$$P = \frac{-3 \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}}{-2} = \begin{pmatrix} -9 \\ 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ 10 \\ -8 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 4 \end{pmatrix}$$

$$P = 4i - 5j + 4k$$

Jawaban: D

7]  $\vec{a} = (3, 4, k_1)$  dan  $\vec{b} = (6, 1, k_2)$

$$|\vec{a}-\vec{b}| = \sqrt{34}, k_1, k_2 = 3$$

$$\vec{a}-\vec{b} = \begin{pmatrix} 3 \\ 4 \\ k_1 \end{pmatrix} - \begin{pmatrix} 6 \\ 1 \\ k_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ k_1-k_2 \end{pmatrix}$$

$$|\vec{a}-\vec{b}| = \sqrt{(-3)^2 + 3^2 + (k_1-k_2)^2}$$

$$\sqrt{34} = \sqrt{9+9+k_1^2-2k_1k_2+k_2^2}$$

$$34 = 18 + k_1^2 - 2(3) + k_2^2$$

$$16 = k_1^2 + k_2^2 - 6$$

$$22 = k_1^2 + k_2^2$$

Jawaban: C

8]  $\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 6 \end{pmatrix}$

$$\vec{AC} = \vec{C} - \vec{A} = \begin{pmatrix} 6 \\ m \\ -6 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ m-5 \\ -6 \end{pmatrix}$$

$$|\vec{AB}| = |\vec{AC}|$$

$$\sqrt{25+1+36} = \sqrt{1+(m-5)^2+36}$$

$$25 = m^2 - 10m + 25$$

$$0 = m^2 - 10m$$

$$0 = m(m-10)$$

$$m=0 \text{ atau } m=10$$

Jawaban: D

$$9 \quad \vec{OA} = i + j + 2k \quad ; \quad \vec{OB} = i + 2j + 3k$$

$$|\vec{PA}| = |\vec{OB}|$$

$$\Rightarrow \vec{AB} = \vec{OB} - \vec{OA}$$

$$= (1, 2, 3) - (1, 1, 2) = (0, 1, 1)$$

$$\Rightarrow \vec{AP} = k \cdot \vec{AB}$$

$$= k (0, 1, 1)$$

$$= (0, k, k)$$

$$\Rightarrow |\vec{AP}| = |\vec{OB}|$$

$$\sqrt{k^2 + k^2} = \sqrt{1^2 + 2^2 + 3^2}$$

$$2k^2 = 1 + 4 + 9$$

$$2k^2 = 14$$

$$k^2 = 7$$

$$k = \pm\sqrt{7}$$

$$\Rightarrow \vec{OA} \cdot \vec{AP}$$

$$= (1, 1, 2) \cdot \sqrt{7} \cdot (0, 1, 1)$$

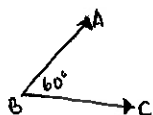
$$= \sqrt{7} (0 + 1 + 2)$$

$$= 3\sqrt{7}$$

Jawaban : C

$$10 \quad \vec{OA} = i + k \quad ; \quad \vec{OB} = j + k \quad ; \quad \vec{OC} = mj + 4k$$

$$\angle ABC = 60^\circ$$



$$\Rightarrow \vec{BA} = \vec{OA} - \vec{OB}$$

$$= (1, 0, 1) - (0, 1, 1)$$

$$= (1, -1, 0)$$

$$\Rightarrow \vec{BC} = \vec{OC} - \vec{OB}$$

$$= (0, m, 4) - (0, 1, 1)$$

$$= (0, m-1, 3)$$

$$\Rightarrow |\vec{BA}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$|\vec{BC}| = \sqrt{(m-1)^2 + 3^2} = \sqrt{m^2 - 2m + 10}$$

$$\Rightarrow \vec{BA} \cdot \vec{BC} = |\vec{BA}| |\vec{BC}| \cos 60^\circ$$

$$(1, -1, 0) \cdot (0, m-1, 3) = \sqrt{2} (\sqrt{m^2 - 2m + 10}) \cdot \frac{1}{2}$$

$$(-m+1) = \left( \sqrt{2} \sqrt{m^2 - 2m + 10} \cdot \frac{1}{2} \right)^2$$

$$1 - 2m + m^2 = \frac{1}{2} \cdot m^2 - 2m + 10$$

$$2 - 4m + 2m^2 = m^2 - 2m + 10$$

$$\Rightarrow m^2 - 2m - 8 = 0$$

$$(m-4)(m+2) = 0$$

$$m = 4 \quad m = -2$$

$$\Rightarrow (m_1 + m_2)^2 = (4 - 2)^2 = 4$$

Jawaban : B

$$11 \quad \vec{a} = 2i - 3j + 5k \quad , \quad \vec{b} = -3i - 5j + 2k$$

$$\Rightarrow \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$= \frac{(-6 + 15 + 10)}{\sqrt{4+9+25} \cdot \sqrt{9+25+4}} = \frac{19}{\sqrt{38} \cdot \sqrt{38}} = \frac{19}{38} = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\Rightarrow \tan 60^\circ = \sqrt{3}$$

Jawaban : E

$$12 \quad |\vec{p}| = 10 \quad ; \quad |\vec{q}| = 6 \quad ; \quad \theta = 60^\circ$$

$$P \cdot Q = |\vec{p}| |\vec{q}| \cos \theta$$

$$= 10 \cdot 6 \cdot \cos 60^\circ$$

$$= 60 \cdot \frac{1}{2} = 30$$

$$\Rightarrow |\vec{p} - \vec{q}|^2 = |\vec{p}|^2 - 2PQ + |\vec{q}|^2$$

$$= 10^2 - 2 \cdot 30 + 6^2$$

$$= 100 - 60 + 36$$

$$= 76$$

$$|\vec{p} - \vec{q}| = \sqrt{76} = 2\sqrt{19}$$

Jawaban : B

$$\begin{aligned} \boxed{13} \quad g &= AB = B - A = (4, 1, -1) - (2, 4, -2) \\ &= (2, -3, 1) \end{aligned}$$

$$\begin{aligned} h &= PQ = Q - P = (8, 2, 1) - (7, 0, 2) \\ &= (1, 2, -3) \end{aligned}$$

$$|g| = \sqrt{4+9+1} = \sqrt{14}$$

$$|h| = \sqrt{1+4+9} = \sqrt{14}$$

$$\Rightarrow \cos \theta = \frac{g \cdot h}{|g| |h|} = \frac{(2-6-3)}{\sqrt{14} \cdot \sqrt{14}} = \frac{-7}{14} = -\frac{1}{2}$$

$$\theta = 120^\circ$$

Jawaban: E

$$\boxed{14} \quad \bar{A} (2k, k+1); \bar{B} (4k, k+5), \bar{C} (10+k, 0)$$

$$\bar{T} = \frac{\bar{A} + \bar{B}}{2} = \frac{(6k, 2k+6)}{2} = (3k, k+3)$$

$$\begin{aligned} \bar{CT} &= \bar{T} - \bar{C} = (3k, k+3) - (10+k, 0) \\ &= (2k-10, k+3) \end{aligned}$$

$$|\bar{CT}| = |0T|$$

$$\sqrt{(2k-10)^2 + (k+3)^2} = \sqrt{(3k)^2 + (k+3)^2}$$

$$4k^2 - 40k + 100 + k^2 + 6k + 9 = 9k^2 + k^2 + 6k + 9$$

$$0 = 5k^2 + 40k - 100$$

$$(5k-10)(k+10) = 0$$

$$\begin{aligned} k &= \frac{10}{5} & k &= -10 \\ &= 2 \end{aligned}$$

$\therefore$  nilai k positif adalah 2

Jawaban: B

$$\boxed{15} \quad A = (-7, 1, -5)$$

$$B = (2, 2, -2)$$

$$\Rightarrow |A| = \sqrt{49+1+25} = \sqrt{75} = 5\sqrt{3}$$

$$|B| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$

$$\Rightarrow |A| - |B| = 5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$$

Jawaban: C

$$\boxed{16} \quad A(6, -2, -6); B(3, 4, 6); C(9, x, y)$$

$$\Rightarrow \bar{AB} = n \cdot \bar{AC}$$

$$\bar{B} - \bar{A} = n (\bar{C} - \bar{A})$$

$$\begin{pmatrix} -3 \\ 6 \\ 12 \end{pmatrix} = n \begin{pmatrix} 3 \\ x+2 \\ y+6 \end{pmatrix}$$

$$\Rightarrow -3 = 3n$$

$$-1 = n$$

$$\Rightarrow (x+2)(-1) = 6$$

$$x+2 = -6$$

$$x = -8$$

$$\Rightarrow (y+6)(-1) = 12$$

$$y+6 = -12$$

$$y = -18$$

$$\Rightarrow x - y = -8 - (-18) = 10$$

Jawaban: D

$$\boxed{17} \quad \bar{a} = (x, 4, 7); \bar{b} = (6, y, 14)$$

$$\bar{a} = n \cdot \bar{b}$$

$$\Rightarrow 7 = n \cdot 14$$

$$\frac{1}{2} = n$$

$$\begin{pmatrix} x \\ 4 \\ 7 \end{pmatrix} = n \begin{pmatrix} 6 \\ y \\ 14 \end{pmatrix}$$

$$\Rightarrow x = 6n$$

$$= 6\left(\frac{1}{2}\right)$$

$$= 3$$

$$\Rightarrow yn = 4$$

$$y = \frac{4}{n} = \frac{4}{\frac{1}{2}} = 8$$

$$\Rightarrow (x-y)^2 = (3-8)^2 = 25$$

Jawaban: C

$$\boxed{18} \quad a = (-1, 2, 2) ; b = (3, -5, 2)$$

$$c = (0, 1, 8)$$

$$\Rightarrow c = pa + qb$$

$$\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix} = \begin{pmatrix} -p \\ 2p \\ 2p \end{pmatrix} + \begin{pmatrix} 3q \\ -5q \\ 2q \end{pmatrix}$$

$$\Rightarrow 2p - 5q = 1$$

$$2p + 2q = 8$$

$$-7q = -7$$

$$q = 1$$

$$p = 3$$

$$\Rightarrow (p+q)^2 = (3+1)^2$$

$$= 4^2$$

$$= 16$$

Jawaban : D

$$\boxed{19} \quad \vec{OA} = (2, -2, 4) ; \vec{OB} = (-2, 4, -2)$$

$$\Rightarrow \vec{OP} = 2\vec{OA}$$

$$= 2(2, -2, 4)$$

$$= (4, -4, 8)$$

$$\Rightarrow \vec{OQ} = -3\vec{OB}$$

$$= -3(-2, 4, -2)$$

$$= (6, -12, 6)$$

$$\Rightarrow \vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (6, -12, 6) - (4, -4, 8)$$

$$= (2, -8, -2)$$

$$\vec{PQ} = 2i - 8j - 2k$$

Jawaban : D

$$\boxed{20} \quad |\vec{a}| = 4 ; |\vec{b}| = 2 ; \vec{a} \cdot (\vec{a} - \vec{b}) = 12$$

$$\Rightarrow |\vec{a}|^2 - \vec{a} \cdot \vec{b} = 12$$

$$16 - \vec{a} \cdot \vec{b} = 12$$

$$\vec{a} \cdot \vec{b} = 4$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$$

$$= 4^2 + 2(4) + 2^2$$

$$= 16 + 8 + 4$$

$$= 28$$

$$|\vec{a} + \vec{b}| = \sqrt{28} = 2\sqrt{7}$$

Jawaban : B

$$\boxed{21} \quad \vec{a}(-1, 1, 2) ; \vec{b}(x, -2, 3)$$

$$\Rightarrow (\vec{a} + \vec{b}) = (x-1, -1, 5)$$

$$\Rightarrow (\vec{b} - \vec{a}) = (x+1, -3, 1)$$

$$(\vec{a} + \vec{b}) \cdot (\vec{b} - \vec{a}) = 11$$

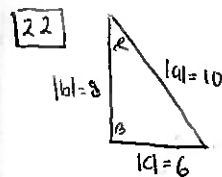
$$(x-1)(x+1) + (-1)(-3) + (5)(1) = 11$$

$$x^2 - 1 + 3 + 5 = 11$$

$$x^2 = 4$$

$$x = \pm 2$$

Jawaban : D



$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\times \cos \theta = \cos 90^\circ = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} = |\vec{a}| |\vec{b}| \cos \alpha + |\vec{b}| |\vec{c}| \cos \beta$$

$$= 8 \cdot 10 \left( \frac{8}{10} \right) + 8 \cdot 6 \cdot 0$$

$$= 64$$

Jawaban : A

$$\boxed{23} \quad \vec{p} \cdot \vec{p} = |\vec{p}|^2 = 3 ; |\vec{q}| = 4$$

$$\Rightarrow |\vec{p} + \vec{q}|^2 = |\vec{p}|^2 + |\vec{q}|^2 + 2|\vec{p}| |\vec{q}| \cos 30^\circ$$

$$= 3 + 16 + 2(\sqrt{3})(4) \frac{1}{2} \sqrt{3}$$

$$= 19 + 12$$

$$= 31$$

$$|\vec{p} + \vec{q}| = \sqrt{31}$$

$$\Rightarrow |\vec{p} - \vec{q}|^2 = |\vec{p}|^2 + |\vec{q}|^2 - 2|\vec{p}| |\vec{q}| \cos 30^\circ$$

$$= 3 + 16 - 2(\sqrt{3})(4) \frac{1}{2} \sqrt{3}$$

$$= 19 - 12$$

$$= 7$$

$$|\vec{p} - \vec{q}| = \sqrt{7}$$

$$\Rightarrow |\vec{p} + \vec{q}| |\vec{p} - \vec{q}| = \sqrt{31} \cdot \sqrt{7}$$

$$= \sqrt{217}$$

Jawaban : E

[24]  $BM : MC = 1 : 3$  ;  $\overrightarrow{BM} = \vec{u}$  ;  $\overrightarrow{CN} = \vec{v}$

$MD : DA = 1 : 4$   $\overrightarrow{BD} = \frac{1}{n} \overrightarrow{BN}$

$\Rightarrow \frac{BM}{MC} = \frac{1}{3}$

$MC = 3BM$

$\Rightarrow \frac{MD}{DA} = \frac{1}{4}$

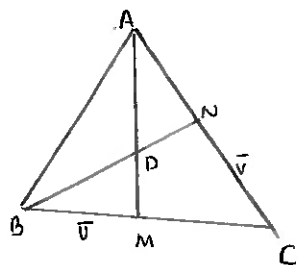
$DA = 4MD$

$\Rightarrow MA = MD + DA$   
 $= 5MD$

$\Rightarrow MA = MC + CA$

$5MD = 3\vec{u} + 2\vec{v}$

$MD = \frac{1}{5} (3\vec{u} + 2\vec{v})$



$\Rightarrow \overrightarrow{BD} = \overrightarrow{BM} + \overrightarrow{MD}$   
 $= \vec{u} + \left( \frac{3}{5}\vec{u} + \frac{2}{5}\vec{v} \right)$   
 $= \frac{8}{5}\vec{u} + \frac{2}{5}\vec{v}$

$\Rightarrow \overrightarrow{BN} = \overrightarrow{BC} + \overrightarrow{CN}$   
 $= 4\vec{u} + \vec{v}$

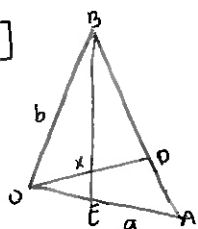
$\Rightarrow \overrightarrow{BD} = \frac{1}{n} \overrightarrow{BN}$

$\frac{8}{5}\vec{u} + \frac{2}{5}\vec{v} = \frac{1}{n} (4\vec{u} + \vec{v})$

$\Rightarrow \frac{1}{n} = \frac{2}{5} \Rightarrow n = \frac{5}{2}$

Jawaban : E

[25]



$\Rightarrow OA = a$

$OB = b$

$AD = \frac{1}{3} AB = \frac{1}{3} (b-a)$

$OE = \frac{1}{3} OA = \frac{1}{3} a$

$\Rightarrow \overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$   
 $= b - \frac{2}{3}(b-a)$   
 $= \frac{2}{3}a + \frac{1}{3}b$

$\Rightarrow \overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$   
 $= -b + \frac{1}{3}a$

$\Rightarrow \overrightarrow{OB} = 0x + xB$

$b = k \cdot \overrightarrow{OD} - l \cdot \overrightarrow{BE}$

$b = k \left( \frac{2}{3}a + \frac{1}{3}b \right) - l \left( -b + \frac{1}{3}a \right)$

$b = \left( \frac{2k}{3} - \frac{l}{3} \right) a + \left( \frac{k}{3} + l \right) b$

$\Rightarrow \frac{2k}{3} - \frac{l}{3} = 0$

$\Rightarrow \frac{k}{3} + l = 1$

$2k - l = 0 \dots (i)$

$k + 3l = 3 \dots (ii)$

dari (i) & (ii)

$\Rightarrow \begin{matrix} 2k - l = 0 & \times 1 \\ k + 3l = 3 & \times 2 \end{matrix}$

$2k - l = 0$

$2k + 6l = 6$

$-7l = -6$

$l = \frac{6}{7}$

$\Rightarrow k = 3 - 3\left(\frac{6}{7}\right)$   
 $= \frac{3}{7}$

$\therefore k = \frac{3}{7}$  dan  $l = \frac{6}{7}$

Jawaban : A

[26]  $\overrightarrow{GM} = 2\vec{u}$  ;  $\overrightarrow{GN} = 2\vec{v}$  ;  $\overrightarrow{GK} = \frac{5}{2} \overrightarrow{LK}$  ;  $\overrightarrow{MN} = k \cdot \overrightarrow{LK}$

$\Rightarrow \overrightarrow{MN} = \overrightarrow{MG} + \overrightarrow{GN}$   
 $= 2\vec{u} + 2\vec{v}$

$\Rightarrow \overrightarrow{HK} = \overrightarrow{HG} + \overrightarrow{GK}$   
 $= 4\vec{u} + \frac{5}{2} \overrightarrow{LK}$

$\Rightarrow \overrightarrow{HN} = \overrightarrow{HM} + \overrightarrow{MN}$   
 $= 2\vec{u} + 2\vec{u} + 2\vec{v}$   
 $= 4\vec{u} + 2\vec{v}$

$\Rightarrow \overrightarrow{HK} = \overrightarrow{HL} + \overrightarrow{LK}$   
 $4\vec{u} + \frac{5}{2} \overrightarrow{LK} = 8\vec{u} + 4\vec{v} + \overrightarrow{LK}$

$\frac{3}{2} \overrightarrow{LK} = 4\vec{u} + 4\vec{v}$

$\overrightarrow{LK} = \frac{8\vec{u} + 8\vec{v}}{3}$

$\Rightarrow \overrightarrow{MN} = k \cdot \overrightarrow{LK}$

$2\vec{u} + 2\vec{v} = k \left( \frac{8}{3}\vec{u} + \frac{8}{3}\vec{v} \right)$

$\Rightarrow 2 = \frac{8k}{3}$

$\frac{6}{8} = k$

$\frac{3}{4} = k$

Jawaban : B

$$\boxed{27} \quad \vec{U} = \vec{a} \quad * \quad \vec{V} = \frac{\vec{q} + \vec{s}}{2}$$

$$\vec{V} - \vec{U} = \vec{a}$$

$$\vec{U} = \frac{\vec{p} + \vec{r}}{2}$$

$$\Rightarrow \vec{PQ} + \vec{PS} + \vec{RQ} + \vec{RS}$$

$$= \vec{Q} - \vec{P} + \vec{S} - \vec{P} + \vec{Q} - \vec{R} + \vec{S} - \vec{R}$$

$$= \vec{q} - \vec{p} + \vec{s} - \vec{p} + \vec{q} - \vec{r} + \vec{s} - \vec{r}$$

$$= 2\vec{q} - 2\vec{p} + 2\vec{s} - 2\vec{r}$$

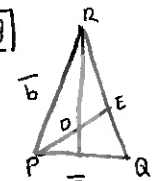
$$= 2(\vec{q} + \vec{s}) - 2(\vec{p} + \vec{r})$$

$$= 2 \cdot 2\vec{V} - 2 \cdot 2\vec{U}$$

$$= 4\vec{V} - 4\vec{U}$$

$$= 4(\vec{V} - \vec{U}) = 4\vec{a}$$

Jawaban: E

$$\boxed{28}$$


$$\Rightarrow \vec{RQ} = \vec{b} - \vec{a}$$

$$\Rightarrow \vec{DE} = \frac{1}{3} \vec{PE}$$

$$\Rightarrow \vec{PE} = \vec{PQ} + \vec{QE}$$

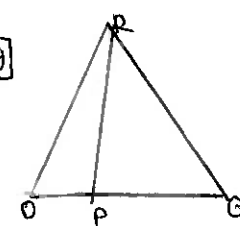
$$= \vec{a} + \frac{1}{2} \vec{QR}$$

$$= \vec{a} + \frac{1}{2} (\vec{b} - \vec{a})$$

$$= \frac{1}{2} \vec{a} + \frac{1}{2} \vec{b}$$

$$\Rightarrow \vec{DE} = \frac{1}{3} \vec{PE} = \frac{1}{3} \left( \frac{1}{2} \vec{a} + \frac{1}{2} \vec{b} \right) = \frac{\vec{a}}{6} + \frac{\vec{b}}{6}$$

Jawaban: A

$$\boxed{29}$$


$$\Rightarrow \cos \angle ROP = \frac{\vec{OR} \cdot \vec{OP}}{|\vec{OR}| |\vec{OP}|}$$

$$\Rightarrow \vec{OR} = \vec{OP} + \vec{PR}$$

$$= 12\vec{a} + 8\vec{a} + 14\vec{b}$$

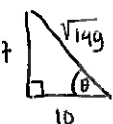
$$= 20\vec{a} + 14\vec{b}$$

$$\Rightarrow |\vec{OR}| = \sqrt{400 + 196} = \sqrt{596} = 2\sqrt{149}$$

$$|\vec{OP}| = \sqrt{144} = 12$$

$$\Rightarrow \cos \angle ROP = \frac{240}{2\sqrt{149} \cdot 12} = \frac{10}{\sqrt{149}}$$

$$\Rightarrow \tan \angle ROP = \frac{7}{10}$$



$$\Rightarrow \cos \angle RQP = \frac{\vec{QR} \cdot \vec{QP}}{|\vec{QR}| |\vec{QP}|}$$

$$\Rightarrow \vec{OP} = 12\vec{a}$$

$$\vec{PR} = 8\vec{a} + 14\vec{b}$$

$$\vec{OR} = \frac{3}{8} \vec{OQ}$$

$$\vec{OQ} = 32\vec{a}$$

$$\Rightarrow \vec{QP} = -20\vec{a}$$

$$|\vec{QP}| = 20$$

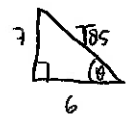
$$\Rightarrow \vec{QR} = \vec{QP} + \vec{PR}$$

$$= -20\vec{a} + 8\vec{a} + 14\vec{b}$$

$$= -12\vec{a} + 14\vec{b}$$

$$|\vec{QR}| = \sqrt{144 + 196} = \sqrt{340} = 2\sqrt{85}$$

$$\Rightarrow \cos \angle RQP = \frac{(240)}{2\sqrt{85} \cdot 20} = \frac{6}{\sqrt{85}}$$



$$\Rightarrow \tan \theta = \frac{7}{6}$$

$$\therefore \tan \angle ROP = \frac{7}{10} \quad \& \quad \tan \angle RQP = \frac{7}{6}$$

Jawaban: E

$$\boxed{30} \quad \vec{OA} = 4\vec{a} \quad \Rightarrow \vec{OP} : \vec{PC} = 3 : 1$$

$$\vec{OC} = 8\vec{c}$$

$$\vec{OP} = \frac{3}{4} \vec{OC} = 6\vec{c}$$

$$\Rightarrow \vec{AB} = \vec{OC} = 8\vec{c}$$

$$\vec{PC} = \frac{1}{4} \vec{OC} = 2\vec{c}$$

$$\Rightarrow L \triangle ABQ = \frac{1}{2} |\vec{BQ}| \cdot |\vec{AB}|$$

$$64 = \frac{1}{2} |\vec{BQ}| \cdot 8$$

$$16 = |\vec{BQ}|$$

$$\Rightarrow \frac{PC}{AB} = \frac{QC}{QB} \Rightarrow |\vec{BQ}| = \left( \frac{4}{3} + 4 \right) |\vec{a}|$$

$$\frac{2}{8} = \frac{x}{x+4}$$

$$16 = \frac{16}{3} |\vec{a}|$$

$$2x + 8 = 8x$$

$$3 = |\vec{a}|$$

$$x = \frac{8}{6} = \frac{4}{3} \vec{a}$$

Jawaban: A

## B. Uraian

11) waku  $t \rightarrow (t, t^2 - 2t, 0)$

a)  $t=1 \rightarrow \vec{P} = (1, -1, 0)$

$\Rightarrow$  vektor posisi  $\vec{P} = \vec{i} - \vec{j}$

$t=2 \rightarrow \vec{Q} = (2, 0, 0)$

$\Rightarrow$  vektor posisi  $\vec{Q} = 2\vec{i}$

b)  $|\vec{P}| = \sqrt{1+1} = \sqrt{2}$

$|\vec{Q}| = \sqrt{2^2} = 2$

$\Rightarrow \cos \theta = \frac{\vec{P} \cdot \vec{Q}}{|\vec{P}| |\vec{Q}|} = \frac{2}{2 \cdot \sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45^\circ$

2)  $A = (-1, 3, 1)$   $C = (0, 2, 5)$   
 $B = (1, 6, 7)$   $D = (1, 4, 10)$

a)  $\vec{AB} = \vec{B} - \vec{A} = (1, 6, 7) - (-1, 3, 1)$   
 $= (2, 3, 6)$

$\vec{CD} = \vec{D} - \vec{C} = (1, 4, 10) - (0, 2, 5)$   
 $= (1, 2, 5)$

b)  $\vec{U} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$  tegak lurus pada  $\vec{AB}$  dan  $\vec{CD}$

$\Rightarrow \vec{U} \cdot \vec{AB} = 0 \quad \Rightarrow \vec{U} \cdot \vec{CD} = 0$

$2x + 3y + 6 = 0$

$x + 2y + 5 = 0$

$2x + 3y = -6 \dots (i)$

$x + 2y = -5 \dots (ii)$

dan (i) dan (ii)

$$\begin{array}{rcl} 2x + 3y = -6 & \times 1 & \\ x + 2y = -5 & \times 2 & \end{array}$$

$$\begin{array}{rcl} 2x + 3y = -6 & & \\ 2x + 4y = -10 & - & \end{array}$$

$-y = 4$

$y = -4$

$\rightarrow x - 8 = -5$   
 $x = 3$

$\therefore x = 3, y = -4, \vec{U} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$

3)  $P = (2, 4, 6); Q = (6, 6, y); R = (14, 2x, -6)$

$\Rightarrow \vec{PQ} = \vec{Q} - \vec{P} = (6, 6, y) - (2, 4, 6)$   
 $= (4, 2, y-6)$

$\Rightarrow \vec{QR} = \vec{R} - \vec{Q} = (14, 2x, -6) - (6, 6, y)$   
 $= (8, 2x-6, -6-y)$

$\Rightarrow P, Q, R$  segaris

$\vec{PQ} = n \cdot \vec{QR} \quad \Rightarrow 4 = n \cdot 8$

$\begin{pmatrix} 4 \\ 2 \\ y-6 \end{pmatrix} = n \cdot \begin{pmatrix} 8 \\ 2x-6 \\ -6-y \end{pmatrix}$

$\frac{1}{2} = n$

$\Rightarrow 2 = \frac{1}{2}(2x-6) \quad \Rightarrow y-6 = \frac{1}{2}(-6-y)$

$4 = 2x-6$

$2y-12 = -6-y$

$10 = 2x$

$3y = 6$

$\boxed{5 = x}$

$\boxed{y = 2}$

$\Rightarrow \vec{PQ} = n \cdot \vec{QR}$

$\therefore \vec{PQ} : \vec{QR} = 1 : 2$

$\vec{PQ} = \frac{1}{2} \vec{QR}$

$\frac{\vec{PQ}}{\vec{QR}} = \frac{1}{2}$

4)  $\vec{U} = 2\vec{i} - \vec{j} + 2\vec{k}$  dan  $\vec{V} = 4\vec{i} + 10\vec{j} - 8\vec{k}$

$(\vec{U} + n\vec{V}) \perp \vec{U}$

$\Rightarrow n\vec{V} = (4n, 10n, -8n)$

$\vec{U} + n\vec{V} = (2, -1, 2) + (4n, 10n, -8n)$   
 $= (2+4n, -1+10n, 2-8n)$

$\Rightarrow (\vec{U} + n\vec{V}) \cdot \vec{U} = 0$

$4+8n+11-10n+4-16n=0$

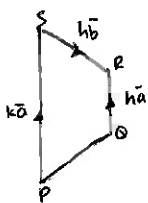
$9-18n=0$

$18n=9$

$n = \frac{9}{18}$

$= \frac{1}{2}$

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$$\Rightarrow \vec{PQ} = 8\vec{a} + \left(\frac{k-6}{2}\right)\vec{b}$$

$$\Rightarrow \vec{PQ} = \vec{PS} + \vec{SR} + \vec{RQ}$$

$$8\vec{a} + \left(\frac{k-6}{2}\right)\vec{b} = k\vec{a} + h\vec{b} - h\vec{a}$$

$$8\vec{a} + \left(\frac{k-6}{2}\right)\vec{b} = (k-h)\vec{a} + h\vec{b}$$

$$\Rightarrow 8 = k-h \quad \dots (i)$$

$$\Rightarrow \frac{k-6}{2} = h$$

$$k-6 = 2h$$

$$k-2h = 6 \quad \dots (ii)$$

$$\begin{aligned} \Rightarrow & \begin{array}{l} k-h=8 \\ k-2h=6 \end{array} \\ & \underline{h=2} \\ & k=10 \end{aligned}$$

a) nilai  $h=2$ , nilai  $k=10$

$$b) L \Delta QRS = \frac{1}{2} QR \cdot t$$

$$40 = \frac{1}{2} \cdot 2 \vec{a} \cdot t$$

$$40 = \vec{a} \cdot t$$

$$L \Delta PQS = \frac{1}{2} PS \cdot t$$

$$= \frac{1}{2} 10 \vec{a} \cdot t$$

$$= 5 \times 40$$

$$= 80 \text{ satuan luas}$$

6 misal panjang proyeksi  $\vec{a}$  pada  $\vec{b} = \vec{c}$

$$\vec{c} = 3|\vec{b}|$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 3|\vec{b}|$$

$$\frac{|\vec{a}||\vec{b}| \cos \alpha}{|\vec{b}|} = 3|\vec{b}|$$

$$|\vec{a}| \cos \alpha = 3|\vec{b}|$$

$$\frac{|\vec{a}|}{|\vec{b}|} = \frac{3}{\cos \alpha}$$

$$a) |\vec{a}| : |\vec{b}| = 3 : \cos \alpha$$

$$b) \alpha = 60^\circ ; |\vec{a}| = 6$$

$$\Rightarrow \frac{|\vec{a}|}{|\vec{b}|} = \frac{3}{\cos \alpha}$$

$$= \frac{3}{\cos 60} = \frac{3}{\frac{1}{2}} = 6$$

$$\Rightarrow |\vec{a}| = 6|\vec{b}|$$

$$6 = 6|\vec{b}|$$

$$1 = |\vec{b}|$$

$$\begin{aligned} \Rightarrow |\vec{a} + \vec{b}| &= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}| \cos 60} \\ &= \sqrt{36 + 1 + 2(1)(6) \cdot \frac{1}{2}} \\ &= \sqrt{37 + 6} \\ &= \sqrt{43} \end{aligned}$$

$$7) \vec{a} = (x, y), \vec{b} = (3, 4), |\vec{a}| = \sqrt{5}$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 2$$

$$\Rightarrow \sqrt{x^2 + y^2} = \sqrt{5}$$

$$x^2 + y^2 = 5$$

$$\frac{3x + 4y}{5} = 2$$

$$y = \sqrt{5 - x^2}$$

$$3x + 4y = 10$$

$$\Rightarrow 3x + 4(\sqrt{5 - x^2}) = 10$$

$$4(\sqrt{5 - x^2}) = 10 - 3x$$

$$\Rightarrow 16(5 - x^2) = 100 - 60x + 9x^2$$

$$80 - 16x^2 = 100 - 60x + 9x^2$$

$$25x^2 - 60x + 20 = 0$$

$$5x^2 - 12x + 4 = 0$$

$$(5x - 2)(x - 2) = 0$$

$$x_1 = \frac{2}{5} \quad x_2 = 2$$

$$\Rightarrow x_1 = \frac{2}{5} \longrightarrow y_1 = \frac{11}{5}$$

$$x_2 = 2 \longrightarrow y_2 = 1$$

$$\therefore \vec{a} = \frac{2}{5}\vec{i} + \frac{11}{5}\vec{j} \text{ dan } \vec{a} = 2\vec{i} + \vec{j}$$



b.)  $\vec{a} = 2\vec{i} + 6\vec{j} + 2\vec{k}$

$$b = -3\hat{i} + x\hat{k}$$

proyeksi  $\vec{a}$  thdp  $\vec{b} = |\vec{p}| = \frac{2}{5}$

$$\Rightarrow |\bar{p}| = \frac{a \cdot b}{|b|}$$

$$\frac{2}{5} = \frac{-6 + 2x}{\sqrt{9+x^2}}$$

$$2\sqrt{9+x^2} = -30 + 10x$$

$$4(9+x^2) = (-30+10x)^2$$

$$36 + 4x^2 = 900 + 600x + 100x^2$$

$$96x^2 + 600x + 884 = 0$$

$$4x^2 - 25x + 36 = 0$$

$$(4x-9)(x-4) = 0$$

$$x_1 = \frac{9}{4} \quad x_2 = 4$$

$$\begin{aligned} \boxed{8} \quad a.) \quad \overline{OP} &= \overline{P} & \Rightarrow \quad \overline{QA} &= \overline{QO} + \overline{OA} \\ OA:AP &= 2:1 & &= -\overline{q} + \frac{2}{3}\overline{p} \\ \Rightarrow \overline{OA} &= \frac{2}{3}\overline{OP} & &= \frac{2}{3}\overline{p} - \overline{q} // \\ &= \frac{2}{3}\overline{p} \end{aligned}$$

b.)  $\overline{AR} : \overline{RQ} = 1 : K$

$$\Rightarrow \overline{RQ} = \frac{k}{k+1} \cdot \overline{QA}$$

$$= \frac{k}{k+1} \left( \frac{2}{3} \bar{p} - \bar{q} \right)$$

$$= \frac{2k}{3k+3} \bar{p} - \frac{k}{k+1} \bar{q}$$

$$= -\frac{2}{9}(2\bar{p} - 3\bar{q}) = \frac{2k}{3k+3}\bar{p} - \frac{k}{k+1}\bar{q}$$

$$\Rightarrow \frac{-4}{9} = \frac{2k}{3k+3}$$

$$-12k - 12 = 18k$$

$$-12 = 30k$$

$$\frac{-12}{30} = k$$

$$-\frac{2}{5} = k$$

9  $\bar{I}_1 = (2, 9)$  ;  $\bar{I}_2 = (-8, 4)$  ;  $\bar{I}_3 = (x, y)$

$$\overline{I_1 I_3} : \overline{I_3 I_2} = 3 : 2$$

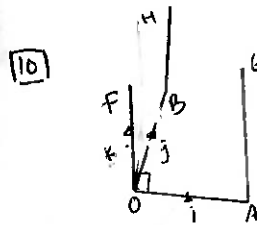
$$\bar{I}_3 = \frac{3 I_2 + 2 I_1}{5} = \frac{3 \begin{pmatrix} -8 \\ 4 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 9 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} -24 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ 18 \end{pmatrix}}{5}$$

$$= \frac{\begin{pmatrix} -20 \\ 36 \end{pmatrix}}{5} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

a.) posisi  $\bar{I}_3 = (-4, 6)$

b.)  $|I_3| = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$



•>  $OF = 10 \text{ m}$

$OA = 4 \text{ m}$

$$Ah = 14 \text{ m}$$

6b = 8 m

a)  $\overline{OF} = 10 \overline{K}$

$$\overline{OB} = \overline{OA} + \overline{AB}$$

$$= 4\bar{i} + 14\bar{k}$$

$$\overline{OH} = \overline{OB} + \overline{BH}$$

$$= 8\vec{j} + 18\vec{k}$$

$$b) \overline{AF} = \overline{AO} + \overline{OF}$$

$$= -4\bar{i} + 10\bar{k}$$

titik tengah  $AF \cdot \frac{AF}{2} = -2\vec{i} + 5\vec{k} = \vec{p}$

$$\Rightarrow \overrightarrow{GH} = \overrightarrow{OH} - \overrightarrow{OG} = (0, 8, 18) - (4, 0, 14) = (-4, 8, 4)$$

$$\Rightarrow \vec{p} - \vec{qH} = \vec{qH} - \vec{p} = (-4, 8, 4) - (-2, 0, 5) = (-2, 8, -1)$$

$$|\vec{p}| = \sqrt{4+64+1}$$
$$= \sqrt{69}$$

∴ Panjang garis adalah  $\sqrt{69}$  m

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