

Uji Pemahaman Halaman 57

A. Pilihan Ganda

1. Pasangan matriks bisa dijumlahkan jika kedua matriks memiliki jumlah baris dan jumlah kolom yang sama sehingga

- $\begin{pmatrix} -2 \\ 3/2 \end{pmatrix}$ yang memiliki 2 baris, 1 kolom dan $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$ yang memiliki 3 baris & 1 kolom

⇒ tidak bisa dijumlahkan karena memiliki jumlah baris yang berbeda.

- $\begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix}$ memiliki 2 baris & 2 kolom

$$\begin{pmatrix} 1 & -3 & 6 \\ 1 & 5 & -7 \\ 1 & 3 & 9 \end{pmatrix} \text{ memiliki } \underline{3 \text{ baris}} \text{ & } \underline{3 \text{ kolom.}}$$

⇒ tidak bisa dijumlahkan karena memiliki jumlah baris (atau kolom) yang berbeda.

- $(1 \ -2 \ 0)$ memiliki 1 baris & 3 kolom, $(3 \ -\frac{1}{2} \ 2)$ memiliki 1 baris & 3 kolom.

⇒ Bisa dijumlahkan

- $(4 \ 3)$ memiliki 1 baris & 2 kolom, $(4 \ 3 \ 0)$ memiliki 1 baris & 3 kolom

⇒ Tidak bisa dijumlahkan karena memiliki jumlah kolom yang berbeda.

- $\begin{pmatrix} 2 & 0 & -5 \\ 3 & -1 & -3 \\ -1 & 4 & 4 \end{pmatrix}$ memiliki 3 baris, 3 kolom

$$\begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix} \text{ memiliki } \underline{2 \text{ baris}}, \underline{2 \text{ kolom.}}$$

⇒ Tidak bisa dijumlahkan karena memiliki jumlah baris (atau kolom) yang berbeda

Jawaban: C

$$\begin{aligned} 2. & \begin{pmatrix} 2 & 3x+y \\ x-y & 4 \end{pmatrix} + \begin{pmatrix} -5 & -y \\ -x & 2 \end{pmatrix} \\ &= \begin{pmatrix} 2+(-5) & (3x+y)+(-y) \\ (x-y)+(-x) & 4+2 \end{pmatrix} \\ &= \begin{pmatrix} -3 & 3x \\ -y & 6 \end{pmatrix} \end{aligned}$$

Jawaban: C

$$\begin{aligned} 3. & \begin{pmatrix} x+6 & -y+2 \\ 2z-4 & 5 \end{pmatrix} + \begin{pmatrix} -x+5 & -y-2 \\ 2z+\frac{3}{2} & a \end{pmatrix} \\ &= \begin{pmatrix} (x+6)+(-x+5) & (-y+2)+(-y-2) \\ (2z-4)+(2z+\frac{3}{2}) & 5+a \end{pmatrix} \\ &= \begin{pmatrix} 11 & -2y \\ 4z-\frac{5}{2} & 5+a \end{pmatrix} = \begin{pmatrix} 11 & -2y \\ \frac{1}{2}(8z-5) & a+5 \end{pmatrix} \end{aligned}$$

Jawaban: A.

$$\begin{aligned} 4. & \begin{pmatrix} 2a+11 & -b+2 \\ 3c+4 & 6 \end{pmatrix} - \begin{pmatrix} -a+11 & -b-3 \\ 3c+b & a-b+6 \end{pmatrix} \\ &= \begin{pmatrix} (2a+11)-(-a+11) & (-b+2)-(-b-3) \\ (3c+4)-(3c+b) & 6-(a-b+6) \end{pmatrix} \\ &= \begin{pmatrix} 3a & 5 \\ 4-b & b-a \end{pmatrix} \end{aligned}$$

Jawaban: E

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5. $A + B = C$

$$\begin{pmatrix} 3x+2 & 4 \\ 4 & y+2 \end{pmatrix} + \begin{pmatrix} -x & -2 \\ 7 & -5y \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ 11 & -2 \end{pmatrix}$$

$$\begin{pmatrix} (3x+2)+(-x) & 4+(-2) \\ 4+7 & (y+2)+(-5y) \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ 11 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 2 \\ 11 & -4y+2 \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ 11 & -2 \end{pmatrix}$$

$$\Rightarrow 2x = 10 \Leftrightarrow x = 5$$

$$-4y+2 = -2 \Leftrightarrow -4y = -4 \Leftrightarrow y = 1$$

$$\therefore x+y = 5+1 = 6$$

Jawaban: E.

6. $P + Q = R$

$$\begin{pmatrix} 11 & 4x+y \\ 2x & -7 \end{pmatrix} + \begin{pmatrix} -6 & x \\ y & 2 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ -8 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 11+(-6) & (4x+y)+x \\ 2x+y & (-7)+2 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ -8 & -5 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 5x+y \\ 2x+y & -5 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ -8 & -5 \end{pmatrix}$$

$$\Rightarrow 5x+y = 10$$

$$2x+y = -8$$

$$3x = 18 \Rightarrow x = 6$$

$$\Rightarrow 2(6)+y = -8$$

$$\Leftrightarrow y = -8-12 = -20$$

$$\therefore x-y = 6-(-20) = 26$$

Jawaban: A.

7. $\begin{pmatrix} x & 5 \\ 1 & y \end{pmatrix} - \begin{pmatrix} 2 & -3 \\ 0 & -3 \end{pmatrix} = \begin{pmatrix} -5 & 8 \\ 1 & -7 \end{pmatrix}$

$$\begin{pmatrix} x-2 & 5-(-3) \\ 1-0 & y-(-3) \end{pmatrix} = \begin{pmatrix} -5 & 8 \\ 1 & -7 \end{pmatrix}$$

$$\Rightarrow x-2 = -5 \Leftrightarrow x = -5+2 = -3$$

$$y-(-3) = -7 \Leftrightarrow y = -7+(-3) = -10$$

$$\Rightarrow x+y = -3+(-10) = -13$$

Jawaban: A.

8. $P - Q - R = \begin{pmatrix} a & 2 \\ 1 & b \\ c & 12 \end{pmatrix}$

$$\begin{pmatrix} 6 & 1 \\ 0 & 3 \\ -4 & 3 \end{pmatrix} - \begin{pmatrix} -2 & -5 \\ 7 & 10 \\ 0 & -9 \end{pmatrix} - \begin{pmatrix} 3 & -4 \\ -8 & 11 \\ -5 & 0 \end{pmatrix} = \begin{pmatrix} a & 2 \\ 1 & b \\ c & 12 \end{pmatrix}$$

$$\begin{pmatrix} 6-(-2)-3 & 1-(-5)-(-4) \\ 0-7-(-8) & 3-10-11 \\ -4-0-(-5) & 3-(-9)-0 \end{pmatrix} = \begin{pmatrix} a & 2 \\ 1 & b \\ c & 12 \end{pmatrix}$$

$$\Rightarrow a = 6-(-2)-3 = 5$$

$$b = 3-10-11 = -18 \Rightarrow a+b+c = 5+(-18)+1$$

$$c = -4-0-(-5) = 1 \quad = 24$$

Jawaban: B.

9.

$$\begin{pmatrix} x+3 & 1 \\ 5 & 1 \end{pmatrix} - \begin{pmatrix} 2y & -2 \\ 1 & y-x \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 3 & 4 \end{pmatrix}^T$$

$$\begin{pmatrix} x+3-2y & 1-(-2) \\ 5-1 & 1-(y-x) \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} x-2y+3 & 3 \\ 4 & x-y+1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 4 & 4 \end{pmatrix}$$

$$\Rightarrow x-2y+3 = 4 \Rightarrow x-2y = 1$$

$$x-y+1 = 4 \Leftrightarrow x-y = 3$$

$$-y = -2$$

$$y = 2$$

$$\Rightarrow x-y = 3 \Rightarrow x-2 = 3 \Leftrightarrow x = 5$$

$$\therefore 2x-y = 2(5)-2 = 8$$

Jawaban: A

10. $\begin{pmatrix} a & 7 \\ -2 & d \end{pmatrix} + \begin{pmatrix} 2 & b \\ c & -8 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ c & -12 \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a+2 & 7+b \\ -2+c & d+(-8) \end{pmatrix} = \begin{pmatrix} 4-a & 3-b \\ c-c & -12-d \end{pmatrix}$$

$$\Rightarrow a+2 = 4-a \Leftrightarrow 2a = 4-2 = 2 \Leftrightarrow a = 1$$

$$7+b = 3-b \Leftrightarrow 2b = 3-7 = -4 \Leftrightarrow b = -2$$

$$-2+c = c-c \Leftrightarrow c = 2$$

$$d+(-8) = -12-d \Leftrightarrow 2d = -4 \Leftrightarrow d = -2$$

$$\Rightarrow a+b+c+d = 1+(-2)+2+(-2) = -1$$

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Jawaban: B.

B. Uraian

1. Pasangan matriks dapat dijumlahkan jika kedua matriks memiliki jumlah baris dan jumlah kolom sama besar

a). $\begin{pmatrix} 4 \\ 8 \end{pmatrix} \Rightarrow \underline{2 \text{ baris}} ; 1 \text{ kolom}$

$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \underline{3 \text{ baris}} ; 1 \text{ kolom}$

\therefore Tidak bisa dijumlah karena kedua matriks memiliki jumlah baris yang berbeda

b). $\begin{pmatrix} 2 & 3 & 8 \end{pmatrix} \Rightarrow 1 \text{ baris} ; 3 \text{ kolom}$

$\begin{pmatrix} 4 & \frac{1}{2} & 5 \end{pmatrix} \Rightarrow 1 \text{ baris} ; 3 \text{ kolom}$

\therefore Bisa dijumlah karena kedua matriks memiliki jumlah baris dan jumlah kolom yang sama.

c). $\begin{pmatrix} 3 & 0 \\ 4 & 8 \end{pmatrix} \Rightarrow \underline{2 \text{ baris}} ; 2 \text{ kolom}$

$\begin{pmatrix} -7 & 1 \\ 5 & 6 \\ 8 & 3 \end{pmatrix} \Rightarrow \underline{3 \text{ baris}} ; 2 \text{ kolom}$

\therefore Tidak bisa dijumlah karena kedua matriks memiliki jumlah baris yang berbeda

d). $\begin{pmatrix} 6 & 5 \\ 2 & 4 \end{pmatrix} \Rightarrow 2 \text{ baris} ; 2 \text{ kolom}$

$\begin{pmatrix} 4 & 1 \\ 0 & 3 \end{pmatrix} \Rightarrow 2 \text{ baris} ; 2 \text{ kolom}$

\therefore Bisa dijumlah karena kedua matriks memiliki jumlah baris dan jumlah kolom yang sama.

e). $\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 2 \text{ baris} ; 1 \text{ kolom}$

$\begin{pmatrix} 5 \\ y \end{pmatrix} \Rightarrow 2 \text{ baris} ; 1 \text{ kolom}$

\therefore Bisa dijumlah karena kedua matriks memiliki jumlah baris dan jumlah kolom yang sama.

2a)(i) $0 + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$= \begin{pmatrix} 0+a & 0+b \\ 0+c & 0+d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\bullet A + 0 = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} a+0 & b+0 \\ c+0 & d+0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\Rightarrow 0 + A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = A + 0$

$\Leftrightarrow 0 + A = A + 0$

(ii) $0 + B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$

$= \begin{pmatrix} 0+2 & 0+(-3) \\ 0+(-1) & 0+5 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$

$\bullet B + 0 = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 2+0 & -3+0 \\ -1+0 & 5+0 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}$

$\Rightarrow 0 + B = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} = B + 0$

$\therefore 0 + B = B + 0$

b) $0 + P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} a & b & c \\ 2 & 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0+a & 0+b & 0+c \\ 0+2 & 0+3 & 0+4 \end{pmatrix} = \begin{pmatrix} a & b & c \\ 2 & 3 & 4 \end{pmatrix}$

$\bullet P + 0 = \begin{pmatrix} a & b & c \\ 2 & 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} a+0 & b+0 & c+0 \\ 2+0 & 3+0 & 4+0 \end{pmatrix} = \begin{pmatrix} a & b & c \\ 2 & 3 & 4 \end{pmatrix}$

$\Rightarrow 0 + P = \begin{pmatrix} a & b & c \\ 2 & 3 & 4 \end{pmatrix} = P + 0 \Rightarrow 0 + P = P + 0$

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$$\begin{aligned} \text{c). (i)} A+B &= \begin{pmatrix} 6 & -5 \\ 1 & -3 \end{pmatrix} + \begin{pmatrix} -6 & 5 \\ -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 6+(-6) & -5+5 \\ 1+(-1) & -3+3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(ii)} B+A &= \begin{pmatrix} -6 & 5 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -5 \\ 1 & -3 \end{pmatrix} \\ &= \begin{pmatrix} -6+6 & 5+(-5) \\ -1+1 & 3+(-3) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} 3. \bullet K+L &= \begin{pmatrix} m & 0 \\ n & p \end{pmatrix} + \begin{pmatrix} q & r \\ s & t \end{pmatrix} \\ &= \begin{pmatrix} m+q & 0+r \\ n+s & p+t \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \bullet L+K &= \begin{pmatrix} q & r \\ s & t \end{pmatrix} + \begin{pmatrix} m & 0 \\ n & p \end{pmatrix} \\ &= \begin{pmatrix} q+m & r+0 \\ s+n & t+p \end{pmatrix} \end{aligned}$$

karena melihat sifat komutatif bil. Asl;

$$K+L = \begin{pmatrix} m+q & 0+r \\ n+s & p+t \end{pmatrix} = \begin{pmatrix} q+m & r+0 \\ s+n & t+p \end{pmatrix} = L+K$$

$$\therefore K+L = L+K.$$

\Rightarrow Operasi penjumlahan pada matriks berlaku sifat komutatif.

$$4. a). (i) P+Q = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$(ii) P+R = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 4 & 1 \end{pmatrix}$$

$$(iii) Q+R = \begin{pmatrix} -2 & 2 \\ -1 & 4 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(iv) Q+P = P+Q = \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$(v) R+P = P+R = \begin{pmatrix} 6 & 3 \\ 4 & 1 \end{pmatrix}$$

$$(vi) R+Q = Q+R = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$b). (i) P+(Q+R) = \begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 3 & 5 \end{pmatrix}$$

$$(ii) (P+Q)+R = \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 3 & 5 \end{pmatrix}$$

$$c). \text{ Misalkan } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}; B = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$C = \begin{pmatrix} w & x \\ y & z \end{pmatrix}.$$

$$\bullet A+(B+C) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \left(\begin{pmatrix} p & q \\ r & s \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix} \right)$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p+w & q+x \\ r+y & s+z \end{pmatrix}$$

$$= \begin{pmatrix} a+(p+w) & b+(q+x) \\ c+(r+y) & d+(s+z) \end{pmatrix}$$

$$= \begin{pmatrix} a+p+w & b+q+x \\ c+r+y & d+s+z \end{pmatrix}$$

$$= \begin{pmatrix} (a+p)+w & (b+q)+x \\ (c+r)+y & (d+s)+z \end{pmatrix}$$

$$= \begin{pmatrix} a+p & b+q \\ c+r & d+s \end{pmatrix} + \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

$$= \left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix} \right) + \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

$$= (A+B) + C.$$

\therefore Penjumlahan matriks berlaku sifat asosiatif.

$$5. a). \begin{pmatrix} 9 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$$

$$b). \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}$$

$$c). \begin{pmatrix} 2a \\ b \end{pmatrix} + \begin{pmatrix} 7a \\ -5b \end{pmatrix} = \begin{pmatrix} 9a \\ -4b \end{pmatrix}$$

$$d). \begin{pmatrix} m \\ n \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} m+1 \\ n+3 \end{pmatrix}$$

$$e). (3 \ 1) + (1 \ 5) = (4 \ 6)$$

$$f). (-2 \ 3) + (6 \ -8) = (4 \ -5)$$

$$g). (3i \ 4j \ -k) + (7i \ -j \ 5k) = (10i \ 3j \ 4k)$$

$$h). \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 4 & 7 \end{pmatrix}$$

$$i). \begin{pmatrix} 4 & 3 \\ 2 & 7 \end{pmatrix} + \begin{pmatrix} 8 & 0 \\ 5 & 9 \end{pmatrix} = \begin{pmatrix} 12 & 3 \\ 7 & 16 \end{pmatrix}$$

$$j). \begin{pmatrix} 2 & 3 & 1 \\ 4 & -1 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 7 & 3 \\ 2 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 8 & 10 & 4 \\ 6 & -2 & 0 \end{pmatrix}$$

$$k). \begin{pmatrix} 2a & b \\ 3a & -b \end{pmatrix} + \begin{pmatrix} a & 5b \\ -4a & b \end{pmatrix} = \begin{pmatrix} 3a & 6b \\ -a & 0 \end{pmatrix}$$

$$1). \begin{pmatrix} x & -y \\ -x & 3y \end{pmatrix} + \begin{pmatrix} 2x & 3y \\ -x & 2y \end{pmatrix} = \begin{pmatrix} 3x & 2y \\ -2x & 5y \end{pmatrix}$$

$$6.a). \begin{pmatrix} x \\ -7 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ y \end{pmatrix} \Leftrightarrow \begin{pmatrix} x-1 \\ -12 \end{pmatrix} = \begin{pmatrix} 5 \\ y \end{pmatrix}$$

$$\Rightarrow x-1 = 5 \Leftrightarrow x = 6 ; y = -12.$$

$$b). (4x \ 7) - (12 \ y) = (2x \ 4)$$

$$\Leftrightarrow (4x-12 \ 7-y) = (2x \ 4)$$

$$\Rightarrow 4x-12 = 2x \Leftrightarrow 2x = 12 \Leftrightarrow x = 6$$

$$\bullet 7-y = 4 \Leftrightarrow y = 3$$

$$c). \begin{pmatrix} 8 & 5 \\ y & -6 \end{pmatrix} - \begin{pmatrix} x & 6 \\ -3 & -6 \end{pmatrix} = \begin{pmatrix} 3x & -1 \\ 5 & 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 8-x & -1 \\ y+3 & 0 \end{pmatrix} = \begin{pmatrix} 3x & -1 \\ 5 & 0 \end{pmatrix}$$

$$\Rightarrow 8-x = 3x \Leftrightarrow 4x = 8 \Leftrightarrow x = 2$$

$$\bullet y+3 = 5 \Leftrightarrow y = 2$$

$$d). \begin{pmatrix} 3x \\ 4y \end{pmatrix} - \begin{pmatrix} 4x \\ -5y \end{pmatrix} = \begin{pmatrix} 14 \\ 11 \end{pmatrix} \Leftrightarrow \begin{pmatrix} -x \\ 9y \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$$

$$\Rightarrow -x = 14 \Leftrightarrow x = -14$$

$$\bullet 9y = 18 \Leftrightarrow y = 2$$

$$7.a). \begin{pmatrix} 13 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \end{pmatrix}$$

$$b). (3 \ 5) - (-1 \ 1) = (4 \ 4)$$

$$c). \begin{pmatrix} -3 \\ 7 \end{pmatrix} - \begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \end{pmatrix}$$

$$d). \begin{pmatrix} -x \\ y \end{pmatrix} - \begin{pmatrix} 3x \\ -4y \end{pmatrix} = \begin{pmatrix} -4x \\ 5y \end{pmatrix}$$

$$e). \begin{pmatrix} a+5 \\ 5-b \end{pmatrix} - \begin{pmatrix} 3 \\ 1+b \end{pmatrix} = \begin{pmatrix} (a+5)-3 \\ (5-b)-(1+b) \end{pmatrix} = \begin{pmatrix} a+2 \\ 4-2b \end{pmatrix}$$

$$f). \begin{pmatrix} a & c \\ b & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & c-f \\ b-g & d-h \end{pmatrix}$$

8.a).

$$A+B-C = \begin{pmatrix} -3 & -1 \\ 5 & 7 \end{pmatrix} + \begin{pmatrix} -3 & 6 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -3+(-3)-4 & -1+6-2 \\ 5+4-(-6) & 7+(-2)-9 \end{pmatrix} = \begin{pmatrix} -10 & 3 \\ 15 & -4 \end{pmatrix}$$

$$b). A-B+C = \begin{pmatrix} -3 & -1 \\ 5 & 7 \end{pmatrix} - \begin{pmatrix} -3 & 6 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 2 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -3-(-3)+4 & -1-6+2 \\ 5-4+(-6) & 7-(-2)+9 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ -5 & 18 \end{pmatrix}$$

$$c). A-B-C = \begin{pmatrix} -3 & -1 \\ 5 & 7 \end{pmatrix} - \begin{pmatrix} -3 & 6 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ -6 & 9 \end{pmatrix} = \begin{pmatrix} -3-(-3)-4 & -1-(6)-2 \\ 5-4-(-6) & 7-(-2)-9 \end{pmatrix} = \begin{pmatrix} -4 & -9 \\ 7 & 0 \end{pmatrix}$$

$$9.(i) A+B = \begin{pmatrix} -5 & 4 \\ 7 & -2 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 3 & 0 \end{pmatrix}$$

(ii) Matriks A dan B sama-sama berbentuk matriks persegi karena memiliki jumlah baris dan jumlah kolom yang sama.

$$10. \begin{pmatrix} 2x \\ 3y \end{pmatrix} + \begin{pmatrix} 4 \\ x \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2x+4 \\ 3y+x \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$$

$$\Rightarrow 2x+4 = 8 \Leftrightarrow 2x = 4 \Leftrightarrow x = 2$$

$$\bullet 3y+x = 11 \Rightarrow 3y+2 = 11 \Leftrightarrow 3y = 9 \Leftrightarrow y = 3$$

$$\therefore x = 2 \text{ dan } y = 3$$

$$11. \begin{pmatrix} 3x+2 & 4 \\ 4 & y+1 \end{pmatrix} + \begin{pmatrix} 2x & -3 \\ 6 & -5y \end{pmatrix} = \begin{pmatrix} 8 & 1 \\ 10 & -6 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 5x+2 & 1 \\ 10 & -4y+1 \end{pmatrix} = \begin{pmatrix} 8 & 1 \\ 10 & -6 \end{pmatrix}$$

$$\Rightarrow 5x+2 = 8 \Leftrightarrow 5x = 6 \Leftrightarrow x = \frac{6}{5}$$

$$\bullet -4y+1 = -6 \Leftrightarrow -4y = -7 \Leftrightarrow y = \frac{7}{4}$$

$$\therefore x = \frac{6}{5} \text{ dan } y = \frac{7}{4}$$

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$$12. \begin{pmatrix} 7 & 4x+y \\ 2x & -3 \end{pmatrix} + \begin{pmatrix} -2 & x \\ y & 2 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ -10 & -1 \end{pmatrix}$$

$$\Rightarrow (4x+y) + x = 0 \Leftrightarrow 5x+y=0$$

$$\begin{aligned} \circ 2x+y &= -10 \Leftrightarrow 2x+y = -10 \quad - \\ \hline 3x &= 10 \Rightarrow x = \frac{10}{3} \end{aligned}$$

$$\Rightarrow 5x+y=0 \Rightarrow y = -5x = -5\left(\frac{10}{3}\right) = -\frac{50}{3}$$

$$\therefore x = \frac{10}{3} \text{ dan } y = -\frac{50}{3}$$

$$13. \begin{pmatrix} 5x-2 & 7 \\ 4y+4 & -3 \end{pmatrix} + \begin{pmatrix} 2y & 2 \\ -3x & -5 \end{pmatrix} = \begin{pmatrix} x+6 & 9 \\ y+7 & -8 \end{pmatrix}$$

$$\Rightarrow (5x-2) + 2y = x+6 \Leftrightarrow 4x+2y=8 \dots (1)$$

$$\circ (4y+4) + (-3x) = y+7 \Leftrightarrow 3y-3x=3 \dots (2)$$

$$\frac{1}{2} \times (1) \Leftrightarrow 2x+y=4$$

$$-\frac{1}{3} \times (2) \Leftrightarrow \frac{x-y}{3x=3} = -1 \quad + \quad \Leftrightarrow x=1$$

$$\circ (2) \Leftrightarrow x-y=-1 \Leftrightarrow y=x+1=1+1=2$$

$$\therefore x=1 \text{ dan } y=2$$

$$(i) x+y=1+2=3$$

$$(ii) x-y=1-2=-1$$

$$14. \begin{pmatrix} 3x+1 \\ y \end{pmatrix} + \begin{pmatrix} 3+2y \\ 3x-4 \end{pmatrix} = \begin{pmatrix} 3x \\ 3y \end{pmatrix} + \begin{pmatrix} 5y-2 \\ 4x \end{pmatrix}$$

$$\Leftrightarrow (3x+1) + (3+2y) = 3x + (5y-2)$$

$$\Leftrightarrow 3y=-6 \Leftrightarrow y=-2$$

$$\circ y + (3x-4) = 3y+4x$$

$$\Leftrightarrow x = -2y-4 = -2(-2)-4 = 0$$

$$(i) y-x = -2-0 = -2$$

$$(ii) x-y = 0-(-2) = 2$$

$$15. A + \begin{pmatrix} 3 & -4 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ 7 & 0 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow A &= \begin{pmatrix} -3 & 2 \\ 7 & 0 \end{pmatrix} - \begin{pmatrix} 3 & -4 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} -3-3 & 2-(-4) \\ 7-0 & 0-5 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 6 \\ 7 & -5 \end{pmatrix} \end{aligned}$$

$$16. (2x \ 3y \ 5z) - (2 \ 2 \ -2) = (x \ 4y \ -4x)$$

$$\circ 2x-2=x \Leftrightarrow x=2$$

$$\circ 3y-2=4y \Leftrightarrow y=-2$$

$$\circ 5z-(-2)=-4x \Leftrightarrow 5z+2=-4(2)$$

$$\Leftrightarrow 5z=-10 \Leftrightarrow z=-2$$

$$\therefore x=2, y=-2, z=-2$$

$$17. \begin{pmatrix} x & -3 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 1 & y \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} -7 & 7 \\ 2 & 7 \end{pmatrix}$$

$$\Rightarrow x-1=-7 \Leftrightarrow x=-6$$

$$\circ -3-y=7 \Leftrightarrow y=-10$$

$$\therefore x=-6 \text{ dan } y=-10$$

$$18. \begin{pmatrix} -3 \\ 3 \\ -7 \end{pmatrix} - \begin{pmatrix} 7x \\ -9y \\ 4z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\Rightarrow -3-7x=x \Leftrightarrow 8x=-3 \Leftrightarrow 4x=-\frac{3}{2}$$

$$\circ 3-(-9y)=y \Leftrightarrow 8y=-3$$

$$\circ -7-4z=z \Leftrightarrow 5z=-7 \Leftrightarrow 3z=-\frac{21}{5}$$

$$\Rightarrow 4x-8y+3z = -\frac{3}{2} - (-3) + \left(-\frac{21}{5}\right)$$

$$= -\frac{15}{10} + \frac{30}{10} - \frac{42}{10}$$

$$= \frac{-27}{10} = -2,7$$

$$19. A + \begin{pmatrix} -2 & 3 & -5 \\ 4 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -7 & 1 & 9 \\ 6 & -3 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} -7 & 1 & 9 \\ 6 & -3 & 0 \end{pmatrix} - \begin{pmatrix} -2 & 3 & -5 \\ 4 & 0 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} -5 & -2 & 14 \\ 2 & -3 & 1 \end{pmatrix}$$

$$20. \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} -3a & 8 \\ 16 & 2d \end{pmatrix} = \begin{pmatrix} 12 & -3b \\ 5c & 4 \end{pmatrix}$$

$$\circ a-(-3a)=12 \Leftrightarrow 4a=12 \Leftrightarrow a=3$$

$$\circ b-8=-3b \Leftrightarrow 4b=8 \Leftrightarrow b=2$$

$$\circ c-16=5c \Leftrightarrow 4c=-16 \Leftrightarrow c=-4$$

$$\circ d-2d=4 \Leftrightarrow -d=4 \Leftrightarrow d=-4$$

$$\Rightarrow a+2b-2c+d = 3+2(2)-2(-4)+(-4) = 11$$

$$21. \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix} - \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ -3 \end{pmatrix} \Rightarrow \begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix} - \begin{pmatrix} -4 \\ 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \\ -4 \end{pmatrix}$$

(6)

$$22. \begin{pmatrix} 2 & 0 & -1 & 7 \end{pmatrix} - X = \begin{pmatrix} -3 & 5 & -3 & 2 \end{pmatrix}$$

$$X = \begin{pmatrix} 2 & 0 & -1 & 7 \end{pmatrix} - \begin{pmatrix} -3 & 5 & -3 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -5 & 2 & 5 \end{pmatrix}$$

$$23. P - Q = \begin{pmatrix} 4 & -1 \\ 3 & 7 \end{pmatrix} - \begin{pmatrix} -1 & 4 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} 5 & -5 \\ -2 & 9 \end{pmatrix}$$

$$29. a). A + B = \begin{pmatrix} 1 & 5 \\ 3 & 2 \\ 4 & 6 \end{pmatrix} + \begin{pmatrix} 4 & -6 \\ 2 & 3 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 5 & 5 \\ 9 & 7 \end{pmatrix}$$

$$b). B - C = \begin{pmatrix} 4 & -6 \\ 2 & 3 \\ 5 & 1 \end{pmatrix} - \begin{pmatrix} 7 & 5 \\ 2 & -3 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} -3 & -11 \\ 0 & 6 \\ 1 & -3 \end{pmatrix}$$

$$c). (A+B) - C = \begin{pmatrix} 5 & -1 \\ 5 & 5 \\ 9 & 7 \end{pmatrix} - \begin{pmatrix} 7 & 5 \\ 2 & -3 \\ 4 & 4 \end{pmatrix} = \begin{pmatrix} -2 & -6 \\ 3 & 8 \\ 5 & 3 \end{pmatrix}$$

$$d). A + (B - C) = \begin{pmatrix} 1 & 5 \\ 3 & 2 \\ 4 & 6 \end{pmatrix} + \begin{pmatrix} -3 & -11 \\ 0 & 6 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} -2 & -6 \\ 3 & 8 \\ 5 & 3 \end{pmatrix}$$

$$25. a). (A+C) + (A+B)$$

$$= \left(\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} \right) + \left(\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 4 & 8 \\ 5 & 2 \end{pmatrix} + \begin{pmatrix} 7 & -1 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 11 & 7 \\ 11 & 9 \end{pmatrix}$$

$$b). A + B - C$$

$$= \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ 6 & 8 \end{pmatrix}$$

$$c). A + B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 6 & 7 \end{pmatrix}$$

$$d). A - C = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -6 \\ 5 & 4 \end{pmatrix}$$

$$e). A - B + C$$

$$= \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} - \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 10 \\ 4 & -2 \end{pmatrix}$$

$$f). B - A - C$$

$$= \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -10 \\ -4 & 2 \end{pmatrix}$$

$$g). A + B + C$$

$$= \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} + \begin{pmatrix} 5 & -2 \\ 1 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 7 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 9 & 6 \\ 6 & 6 \end{pmatrix}$$

$$h). (A+B) - (A+B)$$

$$= \begin{pmatrix} 7 & -1 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 7 & -1 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$26. a). \begin{pmatrix} x \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ 2y \end{pmatrix} + \begin{pmatrix} -1 \\ -11 \end{pmatrix}$$

$$\Rightarrow x = 7 + (-1) = 6 \Rightarrow x = 6$$

$$5 = 2y - 11 \Rightarrow 2y = 16 \Rightarrow y = 8$$

$$\therefore x = 6 \text{ dan } y = 8$$

$$b). \begin{pmatrix} 9 & 3x \\ 2y & 6 \end{pmatrix} + \begin{pmatrix} -6 & -5 \\ -y & y \end{pmatrix} = \begin{pmatrix} 3 & 11 \\ y & 8 \end{pmatrix}$$

$$\Rightarrow 3x + (-5) = 11 \Rightarrow x = \frac{1}{3}$$

$$6 + y = 8 \Rightarrow y = 2$$

$$\therefore x = \frac{1}{3} \text{ dan } y = 2$$

$$c). \begin{pmatrix} 2x & 3y \\ 2y & -x \end{pmatrix} + \begin{pmatrix} -7y & x \\ 3x & -4y \end{pmatrix} = \begin{pmatrix} 11 & 1 \\ 4 & 2 \end{pmatrix}$$

$$\Rightarrow 3y + x = -1$$

$$0 - 4y - x = 2 +$$

$$-y = 1 \Rightarrow y = -1$$

$$\Rightarrow x = -1 - 3y = -1 - 3(-1) = 2$$

$$\therefore x = 2 \text{ dan } y = -1$$

$$27. M = N - L$$

$$\Rightarrow \begin{pmatrix} 3 & -5 \\ 4 & -3 \end{pmatrix} = \begin{pmatrix} 4 & k \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & -7 \\ -3 & 3 \end{pmatrix}$$

$$\Rightarrow -5 = k - (-7) \Rightarrow k = -5 + (-7) = -12$$

$$28. a). X + \begin{pmatrix} 8 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & -1 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} 0 & -2 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -8 & -2 \\ 2 & 0 \end{pmatrix}$$

$$b). \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} + X = \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix}$$

$$\Rightarrow X + \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -6 & -1 \end{pmatrix}$$

$$c). X - \begin{pmatrix} -2 & 3 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ -7 & 4 \end{pmatrix}$$

$$X = \begin{pmatrix} 3 & 9 \\ -7 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 1 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 12 \\ -6 & 10 \end{pmatrix}$$

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$$d). \begin{pmatrix} 7 & -4 \\ 2 & 5 \end{pmatrix} - X = \begin{pmatrix} 3 & -1 \\ -7 & -5 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 3 & -1 \\ -7 & -5 \end{pmatrix} + X = \begin{pmatrix} 7 & -4 \\ 2 & 5 \end{pmatrix}$$

$$\Leftrightarrow X + \begin{pmatrix} 3 & -1 \\ -7 & -5 \end{pmatrix} = \begin{pmatrix} 7 & -4 \\ 2 & 5 \end{pmatrix}$$

$$\Leftrightarrow X = \begin{pmatrix} 7 & -4 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 3 & -1 \\ -7 & -5 \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 9 & 10 \end{pmatrix}$$

$$29.a). X + \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} =$$

$$\Leftrightarrow X = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$$

$$b). (x \ y \ z) - (1 \ 0 \ -5) = (4 \ -3 \ -2)$$

$$\Rightarrow x - 1 = 4 \Leftrightarrow x = 5$$

$$\bullet y - 0 = -3 \Leftrightarrow y = -3$$

$$\bullet z - (-5) = -2 \Leftrightarrow z = -7$$

$$30.a). \begin{pmatrix} p & r \\ q & s \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$$

$$\Rightarrow \bullet p - 2 = 3 \Leftrightarrow p = 5$$

$$\bullet q - 4 = 6 \Leftrightarrow q = 10$$

$$\bullet r - 0 = -1 \Leftrightarrow r = -1$$

$$\bullet s - 1 = -7 \Leftrightarrow s = -6$$

$$b). \begin{pmatrix} -2 & 1 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} -2 & 11 \\ 3 & -1 \end{pmatrix}$$

$$\Rightarrow \bullet -2 - p = -2 \Leftrightarrow p = 0$$

$$\bullet 1 - q = 11 \Leftrightarrow q = -10$$

$$\bullet 0 - r = 3 \Leftrightarrow r = -3$$

$$\bullet 4 - s = -1 \Leftrightarrow s = 5$$

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A. Menghubungkan

$$1. \begin{pmatrix} 3 & -4 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 4 \\ -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \cdot 5 + (-4)(-1) & 3(4) + (-4)(-3) \\ 0(5) + 1(-1) & 0(4) + 1(-3) \end{pmatrix}$$

$$= \begin{pmatrix} 19 & 24 \\ -1 & -3 \end{pmatrix}$$

Jawaban: D.

$$2. \begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 0 & -4 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2(0) + 3(-3) & -2(-4) + 3(1) \\ 4(0) + 1(-3) & 4(-4) + 1(1) \end{pmatrix}$$

$$= \begin{pmatrix} -9 & 11 \\ -3 & -15 \end{pmatrix}$$

Jawaban: A

$$3. \begin{pmatrix} -2 \\ 4 \end{pmatrix} \begin{pmatrix} -1 & 7 \end{pmatrix} = \begin{pmatrix} -2(-1) & -2(7) \\ 4(-1) & 4(7) \end{pmatrix} = \begin{pmatrix} 2 & -14 \\ -4 & 28 \end{pmatrix}$$

Jawaban: E

$$4. \begin{pmatrix} 2 & 4 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2(4) + 4(2) & 2(-1) + 4(3) \\ 5(4) + 1(2) & 5(-1) + 1(3) \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 10 \\ 22 & -2 \end{pmatrix}$$

Jawaban: C.

$$5. \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} -3 & 4 \\ 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 4(-3) + (-1)(2) & 4(4) + (-1)(-1) \\ 2(-3) + 3(2) & 2(4) + 3(-1) \end{pmatrix}$$

$$= \begin{pmatrix} -14 & 17 \\ 0 & 5 \end{pmatrix}$$

Jawaban: B.

B. Pilihan Ganda

1. Pasangan matriks bisa di kalikan jika jumlah kolom pada matriks pertama (kiri) sama dengan jumlah baris pada matriks kedua (kanan).

$$\Rightarrow \begin{pmatrix} -1 \\ 7/2 \end{pmatrix} \rightarrow 2 \text{ kolom} \neq \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} \rightarrow 3 \text{ baris} \left. \vphantom{\begin{pmatrix} -1 \\ 7/2 \end{pmatrix} \rightarrow 2 \text{ kolom}} \right\} \text{Tidak bisa dikalikan.}$$

$$\circ \begin{pmatrix} 1 & 4 \\ 3 & 1 \end{pmatrix} \rightarrow 2 \text{ kolom} \neq \begin{pmatrix} 1 & -2 & 3 \\ 1 & 5 & -3 \\ -1 & 2 & 6 \end{pmatrix} \rightarrow 3 \text{ baris} \left. \vphantom{\begin{pmatrix} 1 & 4 \\ 3 & 1 \end{pmatrix} \rightarrow 2 \text{ kolom}} \right\} \text{Tidak bisa dikalikan}$$

$$\circ \begin{pmatrix} 3 & -1 & 0 \\ -1 & 5 & 1 \end{pmatrix} \rightarrow 3 \text{ kolom} \neq \begin{pmatrix} 1 & -1/5 & -3 \end{pmatrix} \rightarrow 1 \text{ baris} \left. \vphantom{\begin{pmatrix} 3 & -1 & 0 \\ -1 & 5 & 1 \end{pmatrix} \rightarrow 3 \text{ kolom}} \right\} \text{Tidak bisa dikalikan}$$

$$\circ \begin{pmatrix} 2 & 3 \end{pmatrix} \rightarrow 2 \text{ kolom} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 5 & 2 \end{pmatrix} \rightarrow 2 \text{ baris} \left. \vphantom{\begin{pmatrix} 2 & 3 \end{pmatrix} \rightarrow 2 \text{ kolom}} \right\} \text{Bisa dikalikan}$$

$$\circ \begin{pmatrix} 1 & 0 & -5 \\ 4 & -3 & -2 \\ -1 & 2 & 4 \end{pmatrix} \rightarrow 3 \text{ kolom} \neq \begin{pmatrix} 4 & 3 \\ 5 & 1 \end{pmatrix} \rightarrow 2 \text{ baris} \left. \vphantom{\begin{pmatrix} 1 & 0 & -5 \\ 4 & -3 & -2 \\ -1 & 2 & 4 \end{pmatrix} \rightarrow 3 \text{ kolom}} \right\} \text{Tidak bisa dikalikan}$$

Jawaban: D

$$2. PQ = \begin{pmatrix} -4 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & -1 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -4(-3) + 3(1) & -4(-1) + 3(2) \\ -2(-3) + 1(1) & -2(-1) + 1(2) \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 10 \\ 7 & 4 \end{pmatrix}$$

Jawaban: B.

3. $2AB = C$

$$2 \begin{pmatrix} 3 & x \\ 2 & 4 \end{pmatrix} \begin{pmatrix} y & 3 \\ -1 & -x \end{pmatrix} = \begin{pmatrix} 26 & 10 \\ 12 & -4 \end{pmatrix}$$

$$2 \begin{pmatrix} 3y - x & 9 - x^2 \\ 2y - 4 & 6 - 4x \end{pmatrix} = 2 \begin{pmatrix} 13 & 5 \\ 6 & -2 \end{pmatrix}$$

$$\Rightarrow 2y - 4 = 6 \Leftrightarrow 2y = 10 \Leftrightarrow y = 5$$

$$\circ 6 - 4x = -2 \Leftrightarrow 4x = 8 \Leftrightarrow x = 2$$

$$\Rightarrow y - x = 5 - 2 = 3$$

Jawaban: E

4. $3 \begin{pmatrix} -1 \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + k \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$$\Rightarrow 3\left(\frac{1}{3}\right) + 3(0) + k(1) = -1$$

$$\Leftrightarrow 1 + k = -1 \Leftrightarrow k = -2$$

Jawaban: B

5. $\begin{pmatrix} x & 4 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ y \end{pmatrix} = \begin{pmatrix} 51 \\ 7 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} x(5) + 4(y) \\ -1(5) + 3(y) \end{pmatrix} = \begin{pmatrix} 51 \\ 7 \end{pmatrix}$$

$$\Rightarrow -1(5) + 3(y) = 7 \Leftrightarrow 3y = 12 \Leftrightarrow y = 4$$

$$\circ x(5) + 4(y) = 51$$

$$\Rightarrow x(5) + 4(4) = 51$$

$$\Leftrightarrow 5x = 35 \Leftrightarrow x = 7$$

$$\Rightarrow x \cdot y = 7 \cdot 4 = 28$$

Jawaban: A.

6. $B^2 - 2B = \begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix} - 2 \begin{pmatrix} 3 & -1 \\ 4 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 3 \cdot 3 + (-1) \cdot 4 & 3 \cdot (-1) + (-1) \cdot 0 \\ 4 \cdot 3 + 0 \cdot 4 & 4 \cdot (-1) + 0 \cdot 0 \end{pmatrix} - \begin{pmatrix} 6 & -2 \\ 8 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -3 \\ 12 & -4 \end{pmatrix} - \begin{pmatrix} 6 & -2 \\ 8 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 \\ 4 & -4 \end{pmatrix}$$

Jawaban: C

7. $\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & x \\ 2y & 3 \end{pmatrix} \begin{pmatrix} 2 & y \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} x-2 & x-y \\ 4y+3 & 2y^2+3 \end{pmatrix} - \begin{pmatrix} 1 & -2 \\ -4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} x-3 & x-y-2 \\ 4y+7 & 2y^2+2 \end{pmatrix}$$

$$\Rightarrow 1 = x - 3 \Leftrightarrow x = 4$$

$$\circ 3 = 4y + 7 \Leftrightarrow y = -1$$

$$\therefore x = 4 \text{ dan } y = -1$$

Jawaban: E.

8. $P \cdot Q = I$

$$\begin{pmatrix} x & y \\ -3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -3 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x-3y & 3x-5y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow 2x - 3y = 1 \Leftrightarrow 6x - 9y = 3$$

$$\circ 3x - 5y = 0 \Leftrightarrow \underline{6x - 10y = 0}$$

$$y = 3$$

$$\Rightarrow 3x - 5y = 0 \Leftrightarrow 3x = 5y = 5 \cdot 3$$

$$\Leftrightarrow x = 5$$

$$\therefore x^2 - y^2 = 5^2 - 3^2 = 25 - 9 = 16$$

(10) Jawaban: B.

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$$9. \begin{pmatrix} 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} x & y \\ 1 & 0 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \end{pmatrix}$$

$$(3x+1+1 \quad 3y+0-3) \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \end{pmatrix}$$

$$(-(3x+1+1) + 2(3y-3)) = \begin{pmatrix} 7 \end{pmatrix}$$

$$\Rightarrow -(3x+1+1) + 2(3y-3) = 7$$

$$\Leftrightarrow -(3x+2) + 6y - 6 = 7$$

$$\Leftrightarrow -3x - 2 + 6y - 6 = 7$$

$$\Leftrightarrow 6y - 3x = 15 \dots (1)$$

$$\circ x : y = 3 : 2 \Leftrightarrow 3y = 2x \Leftrightarrow 6y = 4x$$

$$(1) \Rightarrow 6y - 3x = 15$$

$$\Rightarrow 4x - 3x = 15 \Rightarrow x = 15$$

$$\circ 6y = 4x$$

$$\Rightarrow 6y = 4(15) \Leftrightarrow y = \frac{60}{6} = 10$$

$$\therefore x = 15 \text{ dan } y = 10$$

Jawaban: A.

$$10. f(p) = p^2 - 3p$$

$$\Leftrightarrow \begin{pmatrix} -(2a+b) & 3 \\ -15 & a+2b \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -5 & 4 \end{pmatrix}$$

$$- 3 \begin{pmatrix} 2 & 1 \\ -5 & 4 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -2a-b & 3 \\ -15 & a+2b \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ -30 & 11 \end{pmatrix} - \begin{pmatrix} 6 & 3 \\ -15 & 12 \end{pmatrix}$$

$$\Rightarrow -2a-b = -1-6 = -7 \Leftrightarrow -2a-b = -7$$

$$\circ a+2b = 11-12 = -1 \Leftrightarrow 2a+4b = -2 +$$

$$3b = -9$$

$$\Leftrightarrow b = -3$$

$$\circ a+2b = -1 \Leftrightarrow a = -1-2b$$

$$= -1-2(-3)$$

$$\Rightarrow a^2 - b^2 = 5^2 - (-3)^2 = 25 - 9 = 16$$

Jawaban: B

C. Uraian

$$1. a). 3 \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \cdot 4 \\ 3 \cdot 5 \end{pmatrix} = \begin{pmatrix} 12 \\ 15 \end{pmatrix}$$

$$b). 5 \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \cdot 2 \\ 5 \cdot (-1) \\ 5 \cdot 0 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \\ 0 \end{pmatrix}$$

$$c). 2 \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 2 & 2 \cdot 3 & 2 \cdot 1 \end{pmatrix} = \begin{pmatrix} 4 & 6 & 2 \end{pmatrix}$$

$$d). 3 \begin{pmatrix} -1 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 \cdot (-1) & 3 \cdot 3 \\ 3 \cdot 1 & 3 \cdot 0 \end{pmatrix} = \begin{pmatrix} -3 & 9 \\ 3 & 0 \end{pmatrix}$$

$$e). 2 \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 & 2 \cdot 3 \\ 2 \cdot 5 & 2 \cdot 2 \end{pmatrix} = \begin{pmatrix} 2 & 6 \\ 10 & 4 \end{pmatrix}$$

$$f). -3 \begin{pmatrix} 3 & -2 & 1 \\ -1 & 5 & 0 \end{pmatrix} = \begin{pmatrix} -3 \cdot 3 & -3 \cdot (-2) & -3 \cdot 1 \\ -3 \cdot (-1) & -3 \cdot 5 & -3 \cdot 0 \end{pmatrix} = \begin{pmatrix} -9 & 6 & -3 \\ 3 & -15 & 0 \end{pmatrix}$$

$$2. a). (i) 3A = 3 \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -3 & 15 \end{pmatrix}$$

$$(ii) 4B = 4 \begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 16 & 4 \\ 12 & -12 \end{pmatrix}$$

$$(iii) -5C = -5 \begin{pmatrix} 0 & -2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 10 \\ 5 & -20 \end{pmatrix}$$

$$(iv) 2A+B = 2 \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ -2 & 10 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 1 & 7 \end{pmatrix}$$

$$(v) 3B-2C = 3 \begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} - 2 \begin{pmatrix} 0 & -2 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 12 & 3 \\ 9 & -9 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ -2 & 8 \end{pmatrix} = \begin{pmatrix} 12 & 7 \\ 11 & -17 \end{pmatrix}$$

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$$\begin{aligned} \text{(vi)} \quad 5(A+B) &= 5\left(\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} + \begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix}\right) \\ &= 5\begin{pmatrix} 5 & 1 \\ 2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 25 & 5 \\ 10 & 10 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(vii)} \quad 7B+7C &= 7\begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} + 7\begin{pmatrix} 0 & -2 \\ -1 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 28 & 7 \\ 21 & -21 \end{pmatrix} + \begin{pmatrix} 0 & -14 \\ -7 & 28 \end{pmatrix} \\ &= \begin{pmatrix} 28 & -7 \\ 14 & 7 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{b). (i)} \quad 5A+5B &= 5\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} + 5\begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 0 \\ -5 & 25 \end{pmatrix} + \begin{pmatrix} 20 & 5 \\ 15 & -15 \end{pmatrix} \\ &= \begin{pmatrix} 25 & 5 \\ 10 & 10 \end{pmatrix} \end{aligned}$$

karena $5A+5B = \begin{pmatrix} 25 & 5 \\ 10 & 10 \end{pmatrix} = 5(A+B)$

$\therefore 5A+5B = 5(A+B)$

$$\begin{aligned} \text{(ii)} \quad 7(B+C) &= 7\left(\begin{pmatrix} 4 & 1 \\ 3 & -3 \end{pmatrix} + \begin{pmatrix} 0 & -2 \\ -1 & 4 \end{pmatrix}\right) \\ &= 7\begin{pmatrix} 4 & -1 \\ 2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 28 & -7 \\ 14 & 7 \end{pmatrix} = 7B+7C \end{aligned}$$

$\therefore 7B+7C = 7(B+C)$

$$\text{3. a).} \quad 5\begin{pmatrix} -1 & 0 & 4 & 5 \\ 0 & 3 & -5 & 6 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 20 & 25 \\ 0 & 15 & -25 & 30 \end{pmatrix}$$

b). $-3Q$

$$= -3\begin{pmatrix} -1 & 0 & 4 & 5 \\ 0 & 3 & -5 & 6 \end{pmatrix} = \begin{pmatrix} 3 & 0 & -12 & -15 \\ 0 & -9 & 15 & -18 \end{pmatrix}$$

c). $\frac{1}{2}Q$

$$= \frac{1}{2}\begin{pmatrix} -1 & 0 & 4 & 5 \\ 0 & 3 & -5 & 6 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 & 2 & \frac{5}{2} \\ 0 & \frac{3}{2} & -\frac{5}{2} & 3 \end{pmatrix} \quad (12)$$

d). $-\frac{1}{4}Q$

$$= -\frac{1}{4}\begin{pmatrix} -1 & 0 & 4 & 5 \\ 0 & 3 & -5 & 6 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & 0 & -1 & -\frac{5}{4} \\ 0 & -\frac{3}{4} & \frac{5}{4} & -\frac{3}{2} \end{pmatrix}$$

4. a). $2M+4N$

$$\begin{aligned} &= 2\begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 3 \end{pmatrix} + 4\begin{pmatrix} -3 & 0 & 2 \\ -1 & 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 6 & -2 & 0 \\ 2 & 0 & 6 \end{pmatrix} + \begin{pmatrix} -12 & 0 & 8 \\ -4 & 16 & 4 \end{pmatrix} \\ &= \begin{pmatrix} -6 & -2 & 8 \\ -2 & 16 & 10 \end{pmatrix} \end{aligned}$$

b). $3M-2N$

$$\begin{aligned} &= 3\begin{pmatrix} 3 & -1 & 0 \\ 1 & 0 & 3 \end{pmatrix} - 2\begin{pmatrix} -3 & 0 & 2 \\ -1 & 4 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 9 & -3 & 0 \\ 3 & 0 & 9 \end{pmatrix} - \begin{pmatrix} -6 & 0 & 4 \\ -2 & 8 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 15 & -3 & -4 \\ 5 & -8 & 7 \end{pmatrix} \end{aligned}$$

5. a) (i) $2X = 2\begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 14 & 4 \\ -2 & -4 \end{pmatrix}$

(ii) $3X = 3\begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 21 & 6 \\ -3 & -6 \end{pmatrix}$

(iii) $-4X = -4\begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -28 & -8 \\ 4 & 8 \end{pmatrix}$

(iv) $-X = -\begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -7 & -2 \\ 1 & 2 \end{pmatrix}$

(v) $(-1)X = (-1)\begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -7 & -2 \\ 1 & 2 \end{pmatrix}$

mis. $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$

$\therefore -A = -\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} -a_1 & -a_2 \\ -a_3 & -a_4 \end{pmatrix}$

$\therefore (-1)A = (-1)\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} = \begin{pmatrix} (-1) \cdot a_1 & (-1) \cdot a_2 \\ (-1) \cdot a_3 & (-1) \cdot a_4 \end{pmatrix} = \begin{pmatrix} -a_1 & -a_2 \\ -a_3 & -a_4 \end{pmatrix}$

$\therefore -A = (-1)A$

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$$\begin{aligned} \text{b). (i)} \quad 2x + 3x &= \begin{pmatrix} 14 & 4 \\ -2 & -4 \end{pmatrix} + \begin{pmatrix} 21 & 6 \\ -3 & -6 \end{pmatrix} \\ &= \begin{pmatrix} 35 & 10 \\ -5 & -10 \end{pmatrix} \\ &= 5 \begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} \end{aligned}$$

$$= 5x$$

$$\therefore 2x + 3x = 5x$$

$$\begin{aligned} \text{(ii)} \quad 5x - 2x &= \begin{pmatrix} 35 & 10 \\ -5 & -10 \end{pmatrix} - \begin{pmatrix} 14 & 4 \\ -2 & -4 \end{pmatrix} \\ &= \begin{pmatrix} 21 & 6 \\ -3 & -6 \end{pmatrix} \end{aligned}$$

$$= 3x$$

$$\therefore 5x - 2x = 3x$$

$$\begin{aligned} \text{(iii)} \quad 2x + x &= \begin{pmatrix} 14 & 4 \\ -2 & -4 \end{pmatrix} + \begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 21 & 6 \\ -3 & -6 \end{pmatrix} \end{aligned}$$

$$= 3x$$

$$\therefore 2x + x = 3x$$

$$\begin{aligned} \text{(iv)} \quad 3x - x &= \begin{pmatrix} 21 & 6 \\ -3 & -6 \end{pmatrix} - \begin{pmatrix} 7 & 2 \\ -1 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 14 & 4 \\ -2 & -4 \end{pmatrix} \end{aligned}$$

$$= 2x$$

$$\therefore 3x - x = 2x$$

$$6. \underbrace{\begin{pmatrix} 2 & 0 & -1 \\ 1 & -4 & -2 \end{pmatrix}}_A - \underbrace{\begin{pmatrix} 5 & -2 & -9 \\ -5 & -4 & 2 \end{pmatrix}}_B = \underbrace{\begin{pmatrix} -3 & 2 & 8 \\ 6 & 0 & -4 \end{pmatrix}}_C$$

$$\Leftrightarrow A - B = C$$

$$\Leftrightarrow A - B + B = C + B$$

$$\Leftrightarrow A = C + B$$

$$\Leftrightarrow -C + A = -C + C + B$$

$$\Leftrightarrow B = A - C = \begin{pmatrix} 2 & 0 & -1 \\ 1 & -4 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 2 & 8 \\ 6 & 0 & -4 \end{pmatrix}$$

$$\Leftrightarrow 3N = \begin{pmatrix} 5 & -2 & -9 \\ -5 & -4 & 2 \end{pmatrix}$$

$$\begin{aligned} \therefore N &= \frac{1}{3} \begin{pmatrix} 5 & -2 & -9 \\ -5 & -4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{5}{3} & -\frac{2}{3} & -3 \\ -\frac{5}{3} & -\frac{4}{3} & \frac{2}{3} \end{pmatrix} \end{aligned}$$

$$7. \text{ mis. } A = \begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix}, C = \begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix}$$

$$\Rightarrow A - 6B = 5B - C$$

$$\Leftrightarrow 11B = A + C$$

$$\Leftrightarrow B = \frac{1}{11}(A + C)$$

$$= \frac{1}{11} \left(\begin{pmatrix} 6 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix} \right)$$

$$= \frac{1}{11} \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{4}{11} \\ \frac{2}{11} \\ \frac{4}{11} \end{pmatrix}$$

$$8. xA + yB = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$$

$$\Leftrightarrow x \begin{pmatrix} -1 \\ 3 \end{pmatrix} + y \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} -x + 2y &= -9 \quad (\Rightarrow) -x + 2y = -9 \\ 3x - y &= 4 \quad (\Rightarrow) 6x - 2y = 8 \end{aligned}$$

$$3x - y = 4 \quad (\Rightarrow) \frac{6x - 2y = 8}{5x = -1}$$

$$5x = -1$$

$$x = -\frac{1}{5}$$

$$\Rightarrow 3x - y = 4 \quad (\Rightarrow) y = 3x - 4$$

$$= 3 \left(-\frac{1}{5} \right) - 4$$

$$= -\frac{3}{5} - \frac{20}{5}$$

$$= -\frac{23}{5}$$

$$\therefore x = -\frac{1}{5} \text{ dan } y = -\frac{23}{5}$$

9. a). $2A + X = 2B$

$$\Rightarrow X = 2B - 2A = 2(B - A)$$

$$= 2 \left(\begin{pmatrix} 1 & -4 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \right)$$

$$= 2 \begin{pmatrix} -3 & -6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -6 & -12 \\ 6 & 4 \end{pmatrix}$$

b). $3A + X = 3B$

$$\Rightarrow X = 3B - 3A = 3(B - A)$$

$$= 3 \left(\begin{pmatrix} 1 & -4 \\ 3 & 5 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \right)$$

$$= 3 \begin{pmatrix} -3 & -6 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} -9 & -18 \\ 9 & 6 \end{pmatrix}$$

c). $2X = C \Leftrightarrow X = \frac{1}{2}C$

$$\Rightarrow X = \frac{1}{2} \begin{pmatrix} 2 & -5 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{5}{2} \\ \frac{1}{2} & \frac{7}{2} \end{pmatrix}$$

d). $X - A = 2X \Leftrightarrow X = -A$

$$\Rightarrow X = - \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -4 & -2 \\ 0 & -3 \end{pmatrix}$$

e). $3X - C = X + A \Leftrightarrow 2X = A + C$

$$\Rightarrow X = \frac{1}{2}(A + C)$$

$$= \frac{1}{2} \left(\begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 2 & -5 \\ 1 & 7 \end{pmatrix} \right)$$

$$= \frac{1}{2} \begin{pmatrix} 6 & -3 \\ 1 & 10 \end{pmatrix} = \begin{pmatrix} 3 & -\frac{3}{2} \\ \frac{1}{2} & 5 \end{pmatrix}$$

f). $2(X + A) = B - A$

$$\Leftrightarrow 2X + 2A = B - A$$

$$\Leftrightarrow 2X = B - 3A$$

$$\Leftrightarrow X = \frac{1}{2}(B - 3A)$$

$$= \frac{1}{2} \left(\begin{pmatrix} 1 & -4 \\ 3 & 5 \end{pmatrix} - 3 \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \right)$$

$$= \frac{1}{2} \begin{pmatrix} -11 & -10 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} -\frac{11}{2} & -5 \\ \frac{3}{2} & -2 \end{pmatrix}$$

10. a). $\begin{pmatrix} 3 & 5 \end{pmatrix} \rightarrow 2 \text{ kolom}$ } Bisa
 $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \rightarrow 2 \text{ baris}$ } dikalikan.

$$\Rightarrow \begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = (3(2) + 5(3)) = (21)$$

b). $\begin{pmatrix} 2 \\ 5 \end{pmatrix} \rightarrow 1 \text{ kolom}$ } Bisa
 $\begin{pmatrix} 3 & 4 & 1 \end{pmatrix} \rightarrow 1 \text{ baris}$ } dikalikan

$$\Rightarrow \begin{pmatrix} 2 \\ 5 \end{pmatrix} \begin{pmatrix} 3 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \cdot 3 & 2 \cdot 4 & 2 \cdot 1 \\ 5 \cdot 3 & 5 \cdot 4 & 5 \cdot 1 \end{pmatrix} = \begin{pmatrix} 6 & 8 & 2 \\ 15 & 20 & 5 \end{pmatrix}$$

c). $\begin{pmatrix} 2 \\ -3 \end{pmatrix} \rightarrow 1 \text{ kolom}$ } Tidak
 $\begin{pmatrix} 0 & 6 \\ 5 & -2 \end{pmatrix} \rightarrow 2 \text{ baris}$ } bisa
 ≠ dikalikan.

d). $\begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} \rightarrow 1 \text{ kolom}$ } Tidak
 $\begin{pmatrix} 1 & -3 & 6 \\ 4 & 5 & -1 \end{pmatrix} \rightarrow 2 \text{ baris}$ } bisa
 ≠ dikalikan.

e). $\begin{pmatrix} -5 \\ 2 \end{pmatrix} \rightarrow 1 \text{ kolom}$ } Bisa
 $\begin{pmatrix} 6 & -4 \end{pmatrix} \rightarrow 1 \text{ baris}$ } dikalikan

$$\Rightarrow \begin{pmatrix} -5 \\ 2 \end{pmatrix} \begin{pmatrix} 6 & -4 \end{pmatrix} = \begin{pmatrix} -5 \cdot 6 & -5 \cdot (-4) \\ 2 \cdot 6 & 2 \cdot (-4) \end{pmatrix}$$

$$= \begin{pmatrix} -30 & 20 \\ 12 & -8 \end{pmatrix}$$

11. a). $\begin{pmatrix} 1 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = (1 \cdot 3 + 5 \cdot 4) = (23)$

b). $\begin{pmatrix} 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = (2 \cdot 1 + 4 \cdot 3 + 6 \cdot 2)$
 $= (26)$

c). $\begin{pmatrix} 2p & -5 & 4p \end{pmatrix} \begin{pmatrix} 5 \\ -3p \\ -2 \end{pmatrix} = (2p \cdot 5 + (-5) \cdot (-3p) - 8)$
 $= (17p)$

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$$d). \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 6 \end{pmatrix} = (4 + 6 + 18) = (28)$$

$$e). (a \ b \ c) \begin{pmatrix} k \\ l \\ m \end{pmatrix} = (ak + bl + cm)$$

$$f). (\cos \alpha \ \sin \alpha) \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$$

$$= (\cos^2 \alpha - \sin^2 \alpha \quad \cos \alpha \sin \alpha + \sin \alpha \cos \alpha)$$

$$= (\cos 2\alpha \quad \sin 2\alpha)$$

$$2. a). \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ap + bq \\ ar + bs \end{pmatrix}$$

$$b). \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + 3b \\ 3a + 2b \end{pmatrix}$$

$$c). \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a - 2b \\ 2a + 3b \end{pmatrix}$$

$$3. a). \begin{pmatrix} 7 & 5 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 21 + 25 \\ 6 + 30 \end{pmatrix} = \begin{pmatrix} 46 \\ 36 \end{pmatrix}$$

$$b). \begin{pmatrix} 4 & 5 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 12 + 20 & 4 + 15 \\ 6 + 24 & 2 + 18 \end{pmatrix}$$

$$= \begin{pmatrix} 32 & 19 \\ 30 & 20 \end{pmatrix}$$

$$c). \begin{pmatrix} 3 \\ 5 \end{pmatrix} \begin{pmatrix} 8 & 6 \end{pmatrix} = \begin{pmatrix} 24 & 18 \\ 40 & 30 \end{pmatrix}$$

$$d). \begin{pmatrix} 8 & 9 & 5 & 2 \\ 3 & 7 & 8 & 1 \\ 8 & 6 & 4 & 10 \end{pmatrix} \begin{pmatrix} 50 \\ 60 \\ 45 \\ 40 \end{pmatrix}$$

$$= \begin{pmatrix} 400 + 540 + 225 + 80 \\ 150 + 420 + 360 + 40 \\ 400 + 360 + 180 + 400 \end{pmatrix} = \begin{pmatrix} 1.245 \\ 970 \\ 1.340 \end{pmatrix}$$

$$e). \begin{pmatrix} 1 & 2 & 3 \\ 7 & 5 & 9 \end{pmatrix} \begin{pmatrix} 4 \\ 7 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 + 14 + 18 \\ 28 + 35 + 54 \end{pmatrix} = \begin{pmatrix} 36 \\ 117 \end{pmatrix}$$

$$f). \begin{pmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{pmatrix} \begin{pmatrix} 4 & -1 & 0 \\ 5 & 3 & -3 \\ 6 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 + 10 + 18 & -1 + 6 + 0 & 0 - 6 + 3 \\ 28 + 40 + 54 & -7 + 24 + 0 & 0 - 24 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 32 & 5 & -3 \\ 122 & 17 & -15 \end{pmatrix}$$

$$g). \begin{pmatrix} 4 \\ 5 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 6 \end{pmatrix} = \begin{pmatrix} 4 & 8 & 24 \\ 5 & 10 & 30 \\ 3 & 6 & 18 \end{pmatrix}$$

$$h). \begin{pmatrix} 2 & 0 & -4 & 1 & -4 \\ 1 & 5 & 0 & -1 & 5 \end{pmatrix} \begin{pmatrix} 2 & 5 & 0 \\ -1 & 0 & -6 \\ 3 & -4 & 4 \\ 0 & -2 & -2 \\ 6 & -3 & 7 \end{pmatrix} = A$$

• Kolom ke-1 :

$$\begin{pmatrix} 4 + 0 - 12 + 0 - 24 \\ 2 - 5 + 0 + 0 + 30 \end{pmatrix} = \begin{pmatrix} -32 \\ 27 \end{pmatrix}$$

• Kolom ke-2 :

$$\begin{pmatrix} 10 + 0 + 16 - 2 + 12 \\ 5 + 0 + 0 + 2 - 15 \end{pmatrix} = \begin{pmatrix} 36 \\ -8 \end{pmatrix}$$

• Kolom ke-3 :

$$\begin{pmatrix} 0 + 0 - 16 - 2 - 28 \\ 0 - 30 + 0 + 2 + 35 \end{pmatrix} = \begin{pmatrix} -46 \\ 7 \end{pmatrix}$$

$$\therefore A = \begin{pmatrix} -32 & 36 & -46 \\ 27 & -8 & 7 \end{pmatrix} //$$

$$14. \begin{pmatrix} a & b \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} -12 & 14 \\ 14 & -5 \end{pmatrix}$$

$$\Rightarrow 2a + 4b = -12 \Leftrightarrow a + 2b = -6$$

$$\bullet a - 2b = 14 \Leftrightarrow \underline{a - 2b = 14} -$$

$$4b = -20$$

$$\Leftrightarrow b = -5.$$

$$\Rightarrow a - 2b = 14 \Leftrightarrow a = 14 + 2b = 14 + 2(-5) = 4$$

$$\Rightarrow 2a - b = 2(4) - (-5) = 8 + 5 = 13 //$$

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15. $RS = \begin{pmatrix} 10 & r \\ 14 & s \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix} \begin{pmatrix} r & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 10 & r \\ 14 & s \end{pmatrix}$$

$$\begin{pmatrix} r^2 + 1 & r \\ sr + 2 & s \end{pmatrix} = \begin{pmatrix} 10 & r \\ 14 & s \end{pmatrix}$$

$$\Rightarrow r^2 + 1 = 10 \Leftrightarrow r^2 = 9 \Rightarrow r = \pm 3$$

$$\Rightarrow sr + 2 = 14 \Leftrightarrow sr = 12 \Rightarrow s = \pm 4$$

* tanda r dan s sama

$$\therefore -2r + 2s = \begin{cases} 2 \cdot 3 + 2 \cdot 4 = 14 \\ 2(-3) + 2(-4) = -14 \end{cases}$$

16. a). $AB + BC$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -1 \\ 22 & -3 \end{pmatrix} + \begin{pmatrix} -5 & 1 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 0 \\ 20 & 1 \end{pmatrix}$$

b). $AC + B$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 7 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 23 \\ 2 & 41 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 22 \\ 4 & 41 \end{pmatrix}$$

c). $A(C+B)$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \left(\begin{pmatrix} -1 & 2 \\ 1 & 7 \end{pmatrix} + \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 10 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 34 & -2 \\ 90 & -2 \end{pmatrix}$$

d). A^2C

$$= \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & 18 \\ 18 & 34 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 146 \\ 16 & 279 \end{pmatrix}$$

e). B^3

$$= \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & -4 \\ 8 & -2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 48 & -14 \\ 28 & -8 \end{pmatrix}$$

17. $A^2 = \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 3 & 18 \end{pmatrix}$

$$\Rightarrow A^2 - 3A - 6I$$

$$= \begin{pmatrix} 3 & 6 \\ 3 & 18 \end{pmatrix} - 3 \begin{pmatrix} -1 & 2 \\ 1 & 4 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 6 \\ 3 & 18 \end{pmatrix} - \begin{pmatrix} -3 & 6 \\ 3 & 12 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

18. $M + N$

$$= \left(\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix} \right)$$

$$= \begin{pmatrix} 3 & 1 \\ -1 & -2 \end{pmatrix}$$

$M - N$

$$= \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix} = \begin{pmatrix} 1 & 7 \\ -1 & 8 \end{pmatrix}$$

$M^2 = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 20 \\ -5 & 5 \end{pmatrix}$$

$N^2 = \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 12 \\ 0 & 25 \end{pmatrix}$$

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a). (i) $(M+N)(M-N)$

$$= \begin{pmatrix} 3 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 7 \\ -1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 29 \\ 2 & -23 \end{pmatrix}$$

(ii) $M^2 - N^2$

$$= \begin{pmatrix} 0 & 20 \\ -5 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 12 \\ 0 & 25 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 8 \\ -5 & -20 \end{pmatrix}$$

(iii) $2MN$

$$= 2 \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix} =$$

$$= 2 \begin{pmatrix} 2 & -26 \\ -1 & -12 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -52 \\ -2 & -24 \end{pmatrix}$$

b) (i) $M^2 - N^2 = \begin{pmatrix} -1 & 8 \\ -5 & -20 \end{pmatrix}$

$$(M+N)(M-N) = \begin{pmatrix} 2 & 29 \\ 2 & -23 \end{pmatrix}$$

$$\therefore M^2 - N^2 \neq (M+N)(M-N)$$

(ii) $(M+N)^2 = \begin{pmatrix} 3 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 8 & 1 \\ -1 & 3 \end{pmatrix}$

$$\bullet M^2 + 2MN + N^2$$

$$= \begin{pmatrix} 0 & 20 \\ -5 & 5 \end{pmatrix} + \begin{pmatrix} 4 & -52 \\ -2 & -24 \end{pmatrix} + \begin{pmatrix} 1 & 12 \\ 0 & 25 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -20 \\ -7 & 6 \end{pmatrix}$$

$$\therefore (M+N)^2 \neq M^2 + 2MN + N^2$$

(iii) $(M-N)^2 = \begin{pmatrix} 1 & 7 \\ -1 & 8 \end{pmatrix} \begin{pmatrix} 1 & 7 \\ -1 & 8 \end{pmatrix} = \begin{pmatrix} -6 & 63 \\ -9 & 57 \end{pmatrix}$

$$\bullet M^2 - 2MN + N^2$$

$$= \begin{pmatrix} 0 & 20 \\ -5 & 5 \end{pmatrix} - \begin{pmatrix} 4 & -52 \\ -2 & -24 \end{pmatrix} + \begin{pmatrix} 1 & 12 \\ 0 & 25 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 84 \\ -3 & 54 \end{pmatrix}$$

$$\therefore (M-N)^2 \neq M^2 - 2MN + N^2$$

(iv) $(2M)N$

$$= \left(2 \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} \right) \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 8 \\ -2 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ 0 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -52 \\ -2 & -24 \end{pmatrix}$$

$$= 2MN$$

$$\therefore (2M)N = 2MN$$

19. $\begin{pmatrix} 3a & 2a \\ 2b & 3b \end{pmatrix} \begin{pmatrix} 2a & 3b \\ 3a & 2b \end{pmatrix} = \begin{pmatrix} 12 & c \\ c & 48 \end{pmatrix}$

$$\begin{pmatrix} 12a^2 & 13ab \\ 13ab & 12b^2 \end{pmatrix} = \begin{pmatrix} 12 & c \\ c & 48 \end{pmatrix}$$

a). $a, b, c \in \text{bil. asli}$

$$\Rightarrow 12a^2 = 12 \Leftrightarrow a^2 = 1 \Rightarrow a = 1$$

$$\bullet 12b^2 = 48 \Leftrightarrow b^2 = 4 \Rightarrow b = 2$$

$$\bullet c = 13ab = 13(1)(2) = 26$$

b). $a, b, c \in \text{bil. real}$

$$\Rightarrow 12a^2 = 12 \Leftrightarrow a^2 = 1 \Leftrightarrow a = \pm 1$$

$$\bullet 12b^2 = 48 \Leftrightarrow b^2 = 4 \Leftrightarrow b = \pm 2$$

* tanda a dan b bisa saja berbeda

$$\bullet c = 13ab = 13(\pm 1)(\pm 2) = \pm 26$$

$$\therefore a, b, c = \pm 1, \pm 2, \pm 26$$

20. a) $I A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 8 & 3 \\ 2 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 1 \cdot 8 + 0 \cdot 2 & 1 \cdot 3 + 0 \cdot 4 \\ 0 \cdot 8 + 1 \cdot 2 & 0 \cdot 3 + 1 \cdot 4 \end{pmatrix} = \begin{pmatrix} 8 & 3 \\ 2 & 4 \end{pmatrix}$$

$$\bullet A I = \begin{pmatrix} 8 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \cdot 1 + 3 \cdot 0 & 8 \cdot 0 + 3 \cdot 1 \\ 2 \cdot 1 + 4 \cdot 0 & 2 \cdot 0 + 4 \cdot 1 \end{pmatrix} = \begin{pmatrix} 8 & 3 \\ 2 & 4 \end{pmatrix}$$

b). karena melihat jawaban(a) dapat disimpulkan bahwa

$$A I = I A = A$$

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A. Menghubungkan

$$1. \begin{vmatrix} 5 & 9 \\ 2 & 8 \end{vmatrix} = 5 \cdot 8 - 9 \cdot 2 = 22$$

Jawaban: E

$$2. \begin{pmatrix} k & -12 \\ -7 & 4 \end{pmatrix} = A, A \text{ tidak punya invers}$$

$$\Rightarrow |A| = 4k - (-12)(-7) = 0$$

$$\Leftrightarrow 4k = 12 \cdot 7$$

$$\Leftrightarrow k = 3 \cdot 7 = 21$$

Jawaban: D

$$3. A = \begin{pmatrix} 2x & 0x \\ -3 & x \end{pmatrix}, A \text{ matriks singular}$$

$$\Rightarrow \det(A) = 0$$

$$\Leftrightarrow 2x(-) - 0x(-3) = 0$$

$$\Leftrightarrow 2x^2 + 24x = 0$$

$$\Leftrightarrow x^2 + 12x = 0$$

$$\Leftrightarrow x(x+12) = 0$$

$$\Rightarrow x = 0 \text{ atau } x = -12$$

$$\therefore \text{karena } x_1, x_2 \Rightarrow x_1 = 0; x_2 = -12$$

$$\Rightarrow x_1 - x_2 = 0 - (-12) = 12$$

Jawaban: B.

$$4. A = \begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{2 \cdot 8 - 3 \cdot 5} \begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 8 & a \\ b & 2 \end{pmatrix} \Rightarrow a + b = -8.$$

Jawaban: A.

$$5. B^{-1} = \begin{pmatrix} 5 & -2 \\ 2 & -1 \end{pmatrix}, |B^{-1}| = 5(-1) - (-2) \cdot 2 = -1$$

$$\Rightarrow (B^{-1})^{-1} = B = \frac{1}{-1} \begin{pmatrix} -1 & 2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 2 & -5 \end{pmatrix}$$

$$\Rightarrow x = 2 \Rightarrow 7x = 14$$

Jawaban: C.

B. Pilihan Ganda

$$1. \det \left(\begin{pmatrix} 2 & -3 \\ 5 & -6 \end{pmatrix}^t \right) = \det \left(\begin{pmatrix} 2 & 5 \\ -3 & -6 \end{pmatrix} \right)$$

$$= 2(-6) - 5(-3)$$

$$= 3$$

Jawaban: E

$$2. \begin{vmatrix} x & 4 \\ 2x & 4x \end{vmatrix} = 12 \Leftrightarrow 4x^2 - 8x = 12$$

$$\Leftrightarrow x^2 - 2x = 3$$

$$\Leftrightarrow x^2 - 2x - 3 = 0$$

$$\Leftrightarrow (x-3)(x+1) = 0$$

$$\Rightarrow x = 3 \text{ atau } x = -1$$

Jawaban: E

$$3. \begin{vmatrix} x+2 & 3 \\ x-1 & x \end{vmatrix} = 5$$

$$\Leftrightarrow (x+2)(x) - 3(x-1) = 5$$

$$\Leftrightarrow x^2 + 2x - 3x + 3 = 5$$

$$\Leftrightarrow x^2 - x - 2 = 0$$

$$\Leftrightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = 2 \text{ atau } x = -1$$

Jawaban: D.

$$4. \begin{vmatrix} 3x & 2x \\ -2 & x \end{vmatrix} = \begin{vmatrix} 3x & -2 \\ 5 & 1 \end{vmatrix}$$

$$\Leftrightarrow 3x^2 - (-4x) = 3x - (-10)$$

$$\Leftrightarrow 3x^2 + x - 10 = 0 \dots \dots \dots (1)$$

$$\circ 3 \cdot (-10) = -30 = 6 \cdot (-5)$$

$$\circ 6 + (-5) = 1$$

$$\Rightarrow (1) \Rightarrow 3x^2 + x - 10 = 0$$

$$\Leftrightarrow \left(x + \frac{6}{3}\right) \left(x - \frac{5}{3}\right) = 0$$

$$\Rightarrow x = -2 \text{ atau } x = \frac{5}{3}$$

Jawaban: A.

5. $\det(A) = 3$

$$\Leftrightarrow (2x)(x+5) - 5 \cdot x = 3$$

$$\Leftrightarrow 2x^2 + 10x - 5x = 3$$

$$\Leftrightarrow 2x^2 + 5x - 3 = 0$$

$$\Leftrightarrow (x+3)(2x-1) = 0$$

$$\Rightarrow x = -3 \text{ atau } x = \frac{1}{2}$$

Jawaban: C.

6. $A = \begin{pmatrix} x-3 & 2 \\ x-3 & x \end{pmatrix}$, A matriks singular

$$\Rightarrow \det(A) = 0$$

$$\Leftrightarrow (x-3)x - (x-3)2 = 0$$

$$\Leftrightarrow (x-3)(x-2) = 0$$

$$\Rightarrow x = 3 \vee x = 2.$$

Jawaban: E

7. $\det(A) = 5 \cdot 3 - 2 \cdot 7 = 15 - 14 = 1$

$$\det(B) = 5 \cdot (-3) - 7 \cdot (-2) = -15 + 14 = -1$$

$$\det(C) = 3 \cdot (-5) - (-7) \cdot 2 = -15 + 14 = -1$$

$$\det(D) = 3 \cdot 5 - (-2) \cdot (-7) = 15 - 14 = 1$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix} = D$$

$$B^{-1} = -\begin{pmatrix} -3 & -7 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 7 \\ -2 & -5 \end{pmatrix}$$

$$C^{-1} = -\begin{pmatrix} -5 & -(-7) \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -7 \\ 2 & -3 \end{pmatrix}$$

$$D^{-1} = \begin{pmatrix} 5 & -(-2) \\ -(-7) & 3 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} = A$$

\therefore Pasangan matriks yang saling invers adalah A dan D

Jawaban: B.

8. $MH = \begin{pmatrix} 0 & 1 & 1 \\ -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}$

$$\Rightarrow (MH)^{-1} = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}^{-1}$$

$$= \frac{1}{3 \cdot 3 - 2 \cdot 5} \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix}$$

$$= \frac{1}{-1} \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix}$$

Jawaban: C.

9. $A^{-1} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{-2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

$$= \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

karena $A^{-1} = \begin{pmatrix} a+1 & a+b \\ c+\frac{1}{2}d & a+b-\frac{1}{2}d \end{pmatrix}$

$$\Rightarrow a+1 = -2 \Rightarrow a = -3$$

$$\Rightarrow a+b = 1 \Rightarrow b = 1-a = 1-(-3) = 4$$

$$\Rightarrow a+b-\frac{1}{2}d = -\frac{1}{2} \Rightarrow \frac{1}{2}d = a+b+\frac{1}{2}$$

$$\Leftrightarrow d = 2(a+b)+1$$

$$\Leftrightarrow d = 2(1)+1 = 3$$

$$\Leftrightarrow d = 3$$

$$\Rightarrow c+\frac{1}{2}d = \frac{3}{2} \Rightarrow c = \frac{3}{2} - \frac{1}{2}d = \frac{3}{2} - \frac{3}{2}$$

$$\Leftrightarrow c = 0$$

$$\therefore a+b+cd = -3+4+0+3 = 4$$

Jawaban: E

10. $A^{-1} = \frac{1}{-4} \begin{pmatrix} -2 & 0 \\ 1 & 2 \end{pmatrix}$

$$B^{-1} = \frac{1}{-\frac{3}{2} + \frac{5}{4}} \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{5}{4} & 1 \end{pmatrix} = -4 \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{5}{4} & 1 \end{pmatrix}$$

$$\Rightarrow A^{-1}B^{-1} = \frac{1}{-4} \begin{pmatrix} -2 & 0 \\ 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -\frac{3}{2} & 1 \\ -\frac{5}{4} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}$$

(19)

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$$A^{-1}B^{-1} = C^T$$

$$\Rightarrow C = (A^{-1}B^{-1})^T = \begin{pmatrix} 3 & -2 \\ -4 & 3 \end{pmatrix}^T = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$$

Jawaban: A

C. Uraian

$$1. a). PQ = \begin{pmatrix} 5 & 3 \\ 7 & 4 \end{pmatrix} \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 45 & 37 \\ 62 & 51 \end{pmatrix}$$

$$b). QP = \begin{pmatrix} 6 & 5 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 65 & 38 \\ 53 & 31 \end{pmatrix}$$

$$c). P^{-1} = \frac{1}{5 \cdot 4 - 3 \cdot 7} \begin{pmatrix} 4 & -3 \\ -7 & 5 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 7 & -5 \end{pmatrix}$$

$$d). Q^{-1} = \frac{1}{6 \cdot 4 - 5 \cdot 5} \begin{pmatrix} 4 & -5 \\ -5 & 6 \end{pmatrix} = \begin{pmatrix} -4 & 5 \\ 5 & -6 \end{pmatrix}$$

$$e). (PQ)^{-1} = \begin{pmatrix} 45 & 37 \\ 62 & 51 \end{pmatrix}^{-1} = \frac{1}{45 \cdot 51 - 37 \cdot 62}$$

$$= \frac{1}{45 \cdot 51 - 37 \cdot 62} \begin{pmatrix} 51 & -37 \\ -62 & 45 \end{pmatrix}$$

$$= \frac{1}{2 \cdot 295 - 2 \cdot 294} \begin{pmatrix} 51 & -37 \\ -62 & 45 \end{pmatrix}$$

$$= \begin{pmatrix} 51 & -37 \\ -62 & 45 \end{pmatrix}$$

$$f). Q^{-1}P^{-1} = \begin{pmatrix} -4 & 5 \\ 5 & -6 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ 7 & -5 \end{pmatrix} = \begin{pmatrix} 51 & -37 \\ -62 & 45 \end{pmatrix}$$

$$g). \det(QP) = 65 \cdot 31 - 38 \cdot 53$$

$$= 2015 - 2014$$

$$= 1$$

$$\Rightarrow (QP)^{-1} = \begin{pmatrix} 65 & 38 \\ 53 & 31 \end{pmatrix}^{-1} = \begin{pmatrix} 31 & -38 \\ -53 & 65 \end{pmatrix}$$

$$h). P^{-1}Q^{-1} = \begin{pmatrix} -4 & 3 \\ 7 & -5 \end{pmatrix} \begin{pmatrix} -4 & 5 \\ 5 & -6 \end{pmatrix} = \begin{pmatrix} 31 & -38 \\ -53 & 65 \end{pmatrix}$$

$$2. a). A = \begin{pmatrix} -3 & -4 \\ 7 & 4 \end{pmatrix} \Rightarrow \det(A) = -3 \cdot 4 - (-4) \cdot 7$$

$$= 16$$

$$\Rightarrow A^{-1} = \frac{1}{16} \begin{pmatrix} 4 & 4 \\ -7 & -3 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{7}{16} & -\frac{3}{16} \end{pmatrix}$$

$$b). B = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ -2 & 4 \end{pmatrix} \Rightarrow \det(B) = \frac{1}{4} \cdot 4 - \frac{1}{2} \cdot (-2)$$

$$= 2$$

$$\Rightarrow B^{-1} = \frac{1}{2} \begin{pmatrix} 4 & -\frac{1}{2} \\ 2 & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 2 & -\frac{1}{4} \\ 1 & \frac{1}{8} \end{pmatrix}$$

$$c). C = \begin{pmatrix} -8 & 7 \\ -6 & -5 \end{pmatrix} \Rightarrow \det(C) = (-8)(-5) - 7 \cdot (-6)$$

$$= 82$$

$$\Rightarrow C^{-1} = \frac{1}{82} \begin{pmatrix} -5 & -7 \\ 6 & -8 \end{pmatrix} = \begin{pmatrix} -\frac{5}{82} & -\frac{7}{82} \\ \frac{3}{41} & -\frac{4}{41} \end{pmatrix}$$

$$d). D = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ 2 & 4 \end{pmatrix} \Rightarrow \det(D) = \frac{1}{4} \cdot 4 - \frac{1}{2} \cdot 2 = 0$$

$\therefore D^{-1}$ tidak ada

$$e). E = \begin{pmatrix} x & y \\ y & x \end{pmatrix} \Rightarrow \det(E) = x^2 - y^2$$

$$\Rightarrow E^{-1} = \frac{1}{x^2 - y^2} \begin{pmatrix} x & -y \\ -y & x \end{pmatrix}$$

$$= \begin{pmatrix} \frac{x}{x^2 - y^2} & -\frac{y}{x^2 - y^2} \\ -\frac{y}{x^2 - y^2} & \frac{x}{x^2 - y^2} \end{pmatrix}$$

dimana $x \neq y$.

$$3. \det(A) = 9 \cdot 5 - 22 \cdot 2 = 45 - 44 = 1$$

$$a). A^{-1} = \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix}$$

$$b). AA^{-1} = \begin{pmatrix} 9 & 22 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$c). A^{-1}A = \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix} \begin{pmatrix} 9 & 22 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

d). Hasil dari AA^{-1} dan $A^{-1}A$ menghasilkan matriks identitas yang menunjukkan bahwa invers dari suatu matriks invers adalah matriks itu sendiri

4.a). $Q \begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$

$$\Rightarrow Q = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} \cdot \frac{1}{1} \begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 2 & -5 \\ -3 & 8 \end{pmatrix}$$

$$\Rightarrow Q = \begin{pmatrix} -7 & 19 \\ -16 & 42 \end{pmatrix}$$

b). $\begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} Q = \begin{pmatrix} -5 & 8 \\ -1 & 5 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} Q = \begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} -5 & 8 \\ -1 & 5 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix} Q = \begin{pmatrix} 13 & -3 \\ 4 & -3 \end{pmatrix}$$

$$\Leftrightarrow Q = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 13 & -3 \\ 4 & -3 \end{pmatrix}$$

$$= \frac{1}{4+6} \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 13 & -3 \\ 4 & -3 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 40 & -3 \\ 30 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -\frac{3}{10} \\ 3 & -\frac{9}{10} \end{pmatrix}$$

c). $\begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix} Q_1 = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$

$$\Leftrightarrow Q_1 = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$$

$$= \frac{1}{2-3} \begin{pmatrix} -1 & -3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 14 \\ 8 & 9 \end{pmatrix}$$

d). $Q_2 \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$

$$Q_2 = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \cdot \frac{1}{2-3} \begin{pmatrix} -1 & -3 \\ -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 \\ 8 & 19 \end{pmatrix}$$

$\therefore Q_1 \neq Q_2$

e). $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} Q = \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} Q = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix}$$

$$\Leftrightarrow Q = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}^{-1} \left(\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix} \right)$$

$$= \frac{1}{1} \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -19 & -11 \\ 14 & 8 \end{pmatrix}$$

f). $\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 5 \end{pmatrix} = Q \begin{pmatrix} -1 & 3 \\ -1 & 2 \end{pmatrix}$

$$Q = \left(\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 5 \end{pmatrix} \right) \begin{pmatrix} -1 & 3 \\ -1 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 5 & -4 \\ -1 & -4 \end{pmatrix} \cdot \frac{1}{-2-(-3)} \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -4 \\ -1 & -4 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -11 \\ -6 & 7 \end{pmatrix}$$

5. $P = R - QS \Leftrightarrow QS = R - P$

$$\Leftrightarrow QS \cdot S^{-1} = (R - P) S^{-1}$$

$$\Leftrightarrow Q = (R - P) S^{-1}$$

$$\Rightarrow Q = \left(\begin{pmatrix} 0 & -1 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} -21 & -21 \\ -6 & -5 \end{pmatrix} \right) \begin{pmatrix} 3 & -2 \\ -5 & -8 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 21 & 20 \\ 8 & 6 \end{pmatrix} \cdot \frac{1}{-24-10} \begin{pmatrix} -8 & 2 \\ 5 & 3 \end{pmatrix}$$

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$$\therefore Q = \frac{1}{-34} \begin{pmatrix} 21 & 20 \\ 8 & 6 \end{pmatrix} \begin{pmatrix} -8 & 2 \\ 5 & 3 \end{pmatrix}$$

$$= -\frac{1}{34} \begin{pmatrix} -68 & 102 \\ -34 & 34 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$$

$$6. \begin{pmatrix} 3-a & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} -3 \\ x \end{pmatrix} = \begin{pmatrix} -3 \\ x \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} (3-a) \cdot (-3) + 3x \\ (-1) \cdot (-3) - 2x \end{pmatrix} = \begin{pmatrix} -3 \\ x \end{pmatrix}$$

$$\Rightarrow 03 - 2x = x \Leftrightarrow 3x = 3 \Leftrightarrow x = 1$$

$$\bullet -9 + 3a + 3x = -3$$

$$\Leftrightarrow 3a = -3 + 9 - 3x = 6 - 3(1) = 3$$

$$\Leftrightarrow a = 1$$

$$7. a) \begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix} = (5 \cdot 7 + 3 \cdot 4) = (47)$$

$$b) \begin{pmatrix} a & b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = (61)$$

$$\Leftrightarrow a^2 + b^2 = 61 = 25 + 36 = 5^2 + 6^2$$

Misalkan, a, b adalah bilangan

bulat, maka

$$(a, b) = \{ (5, 6), (5, -6),$$

$$(-5, 6), (-5, -6),$$

$$(6, 5), (6, -5),$$

$$(-6, 5), (-6, -5) \}$$

$$8. a). (i) AB = \begin{pmatrix} 3 & 0 \\ 2 & 7 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 4 & k \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 30 & 7k \end{pmatrix}$$

$$(ii) BA = \begin{pmatrix} 1 & 0 \\ 4 & k \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 12 + 2k & 7k \end{pmatrix}$$

$$(iii) AB = BA$$

$$\Rightarrow 12 + 2k = 30 \Leftrightarrow 2k = 30 - 12 = 18$$

$$\Leftrightarrow k = 9$$

$$b). \det(A) = 21; \det(B) = 9$$

$$\Rightarrow A^{-1} = \frac{1}{21} \begin{pmatrix} 7 & 0 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{2}{21} & \frac{1}{7} \end{pmatrix}$$

$$B^{-1} = \frac{1}{9} \begin{pmatrix} 1 & 0 \\ 4 & 9 \end{pmatrix} = \begin{pmatrix} \frac{1}{9} & 0 \\ \frac{4}{9} & 1 \end{pmatrix}$$

$$9. a). \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 4 \end{pmatrix} = (7 + 8) = (15)$$

$$b). \begin{pmatrix} 2 \\ -3 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 + 3 \\ -3 + 6 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

$$c). x \begin{pmatrix} 15 \\ -8 \end{pmatrix} + 3 \begin{pmatrix} 5 \\ y \end{pmatrix} = \begin{pmatrix} 75 \\ 19 \end{pmatrix}$$

$$\Rightarrow 15x + 15 = 75 \Leftrightarrow 15x = 60 \Leftrightarrow x = 4$$

$$\bullet -8x + 3y = 19$$

$$\Leftrightarrow -32 + 3y = 19 \Leftrightarrow 3y = 51$$

$$\Leftrightarrow y = 17$$

$$10. a). AB$$

$$= \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & x \\ y & -1 \end{pmatrix} = \begin{pmatrix} 2 & 2x \\ 3+y & 3x-1 \end{pmatrix}$$

$$b). B + pA = \begin{pmatrix} 7 & 5 \\ 13 & 2 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} 1 & x \\ y & -1 \end{pmatrix} + p \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 13 & 2 \end{pmatrix}$$

$$\Rightarrow 0 \cdot x + p \cdot 0 = 5 \Leftrightarrow x = 5$$

$$\bullet 1 + p \cdot 2 = 7 \Leftrightarrow 2p = 6 \Leftrightarrow p = 3$$

$$\bullet y + p \cdot 3 = 13 \Leftrightarrow y = 13 - 3p = 13 - 3(3)$$

$$\Leftrightarrow y = 4$$

$$\bullet 2 = -1 + p \cdot 1 = -1 + 3 = 2 \Rightarrow z = 2$$

$$11. a). \begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix}$$

$$\Rightarrow \det(A) = 2 \cdot 8 - 5 \cdot 7 = 16 - 35 = -19$$

$$b). B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 7 & 8 \end{pmatrix}$$

$$\Rightarrow \det(B) = 2 \cdot 8 - 5 \cdot 7 = 16 - 35 = -19$$

$$c). C = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 6 & 3 \end{pmatrix}$$

$$\Rightarrow \det(C) = -2 \cdot 3 - 4 \cdot 6 = -30$$

$$d). D = \begin{pmatrix} 6 & 3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 4 & -2 \end{pmatrix}$$

$$\Rightarrow \det(D) = 3 \cdot (-2) - 6 \cdot 4 = -30$$

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12. a)
$$A = \begin{pmatrix} 3 & 7 \\ 8 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -3 & 5 \\ 2 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 7 \\ 8 & -1 \end{pmatrix} - \begin{pmatrix} -3 & 5 \\ 2 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 2 \\ 11 & -8 \end{pmatrix}$$

$\Rightarrow \det(A) = (6)(-8) - (2)(11) = -70$

b)
$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 6 & -3 \end{pmatrix} - 3 \begin{pmatrix} 2 & 4 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 7 \\ 6 & -3 \end{pmatrix} - \begin{pmatrix} 6 & 12 \\ -3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -5 \\ 9 & -3 \end{pmatrix}$$

$\Rightarrow \det(B) = (-1)(-3) - (-5)(9) = 48$

c)
$$C = \begin{pmatrix} 8 & -6 \\ 10 & 2 \end{pmatrix} + 6 \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -6 \\ 10 & 2 \end{pmatrix} + 6 \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & -6 \\ 10 & 2 \end{pmatrix} + \begin{pmatrix} 6 & 18 \\ -12 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 14 & 12 \\ -2 & 8 \end{pmatrix}$$

3. a)
$$A = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ -2 & 2x+1 \end{pmatrix}$$
, A matriks singular
 $\Rightarrow \det(A) = 0$

$\Rightarrow \frac{1}{4}(2x+1) - \frac{1}{2} \cdot (-2) = 0$

$\Leftrightarrow \frac{1}{2}x + \frac{1}{4} + 1 = 0$

$\Leftrightarrow x = 2 \cdot \left(-\frac{5}{4}\right) = -\frac{5}{2} \Rightarrow x = -\frac{5}{2}$

b)
$$B = \begin{pmatrix} \frac{1}{4} & x \\ x+1 & 8 \end{pmatrix}$$
, B matriks singular
 $\Rightarrow \det(B) = 0$

$\Rightarrow \frac{1}{4} \cdot 8 - x(x+1) = 0$

$\Leftrightarrow -x^2 - x + 2 = 0$

$\Leftrightarrow x^2 + x - 2 = 0$

$\Leftrightarrow (x+2)(x-1) = 0$

$\Rightarrow x = -2 \text{ atau } x = 1$

14. a)
$$AB = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 30 & 20 \\ 15 & 10 \end{pmatrix}$$

$\Rightarrow \det(AB) = 0 \cdot 10 - 20 \cdot 15 = 0$

$$BA = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 16 \\ 16 & 32 \end{pmatrix}$$

$\Rightarrow \det(BA) = 8 \cdot 32 - 16 \cdot 16 = 0$

karena $\det(AB) = 0$, dan

$\det(BA) = 0$

\therefore Matriks AB dan BA adalah matriks singular.

b)
$$AC = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 28 & 0 \\ 14 & 0 \end{pmatrix}$$

$\Rightarrow \det(AC) = 28 \cdot 0 - 0 \cdot 14 = 0$

$$BC = \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 18 & 8 \\ 36 & 16 \end{pmatrix}$$

$\Rightarrow \det(BC) = 18 \cdot 16 - 8 \cdot 36 = 0$

$$CA = \begin{pmatrix} 2 & 4 \\ 6 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 16 \\ 10 & 20 \end{pmatrix}$$

$\Rightarrow \det(CA) = 8 \cdot 20 - 16 \cdot 10 = 0$

$$CB = \begin{pmatrix} 2 & 4 \\ 6 & -2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 30 & 20 \\ 6 & 4 \end{pmatrix}$$

$\Rightarrow \det(CB) = 30 \cdot 4 - 20 \cdot 6 = 0$

karena $\det(AC) = 0$; $\det(BC) = 0$;

$\det(CA) = 0$, dan $\det(CB) = 0$

\therefore Matriks AC, BC, CA, dan CB

adalah matriks singular

15. a)
$$x(x+1) - (x+1)(-4) = 0$$

$\Leftrightarrow (x+1)(x+4) = 0$

$\Leftrightarrow x = -1 \text{ atau } x = -4$

b)
$$\sin x = 2 \Rightarrow 1 \cdot 1 = 0$$

$\Leftrightarrow 2 \sin x = 1$

$\Leftrightarrow \sin x = \frac{1}{2}$

$\Leftrightarrow x = 30^\circ, 150^\circ, \dots$

Uji Pemahaman Halaman 73

$$16.a) \det(A) = 16 \cdot 4 - 7 \cdot 9 = 64 - 63 = 1$$

$$\therefore A^{-1} = \frac{1}{1} \begin{pmatrix} 4 & -7 \\ -9 & 16 \end{pmatrix} = \begin{pmatrix} 4 & -7 \\ -9 & 16 \end{pmatrix}$$

$$b). \det(A^{-1}) = 4 \cdot 16 - (-7)(-9) = 1$$

$$(A^{-1})^{-1} = \frac{1}{1} \begin{pmatrix} 16 & -(-7) \\ -(-9) & 4 \end{pmatrix} = \begin{pmatrix} 16 & 7 \\ 9 & 4 \end{pmatrix}$$

$$17.a). \det(B) = -13 \cdot 8 - (-5)(21)$$

$$= -104 + 105$$

$$= 1$$

$$B^{-1} = \frac{1}{1} \begin{pmatrix} 8 & -(-5) \\ -21 & -13 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -21 & -13 \end{pmatrix}$$

$$b). |B^{-1}| = \det(B^{-1}) = 8 \cdot (-13) - (5)(-21)$$

$$= -104 + 105$$

$$= 1$$

$$18.a). AB = \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$BA = \begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix} \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

b). Berdasarkan jawaban (a), maka

$$AB = BA = I$$

$$c). \circ \det(A) = 7 \cdot 4 - 9 \cdot 3 = 1$$

$$\Rightarrow A^{-1} = \frac{1}{1} \begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix}$$

$$\circ \det(B) = 4 \cdot 7 - (-9)(-3) = 1$$

$$\Rightarrow B^{-1} = \frac{1}{1} \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 3 & 4 \end{pmatrix}$$

d). Berdasarkan pada jawaban (c),

$$\text{maka } A^{-1} = B \text{ dan } B^{-1} = A$$

$$\therefore A = B^{-1} \text{ dan } B = A^{-1}$$

selesai

$$19.a). (i) (PQ)^{-1} = \left(\begin{pmatrix} -3 & 1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \right)^{-1}$$

$$= \begin{pmatrix} -8 & -10 \\ 10 & 12 \end{pmatrix}^{-1}$$

$$= \frac{1}{(-8) \cdot 12 - (-10) \cdot 10} \begin{pmatrix} 12 & 10 \\ -10 & -8 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 12 & 10 \\ -10 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{5}{2} \\ -\frac{5}{2} & -2 \end{pmatrix}$$

$$(ii) QP = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -3 & 1 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ 5 & -3 \end{pmatrix}$$

$$\det(QP) = 7(-3) - (-5) \cdot 5 = 4$$

$$(QP)^{-1} = \frac{1}{4} \begin{pmatrix} -3 & 5 \\ -5 & 7 \end{pmatrix} = \begin{pmatrix} -\frac{3}{4} & \frac{5}{4} \\ -\frac{5}{4} & \frac{7}{4} \end{pmatrix}$$

$$(iii) \det(P) = (-3)(-2) - 1 \cdot 4 = 2$$

$$P^{-1} = \frac{1}{2} \begin{pmatrix} -2 & -1 \\ -4 & -3 \end{pmatrix} = \begin{pmatrix} -1 & -\frac{1}{2} \\ -2 & -\frac{3}{2} \end{pmatrix}$$

$$(iv) \det(Q) = 3 \cdot 2 - 4 \cdot 1 = 2$$

$$Q^{-1} = \frac{1}{2} \begin{pmatrix} 2 & -4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

$$(iv) P^{-1}Q^{-1} = \begin{pmatrix} -1 & -\frac{1}{2} \\ -2 & -\frac{3}{2} \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -1 + \frac{1}{4} & 2 - \frac{3}{4} \\ -2 + \frac{3}{4} & 4 - \frac{9}{4} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{3}{4} & \frac{5}{4} \\ -\frac{5}{4} & \frac{7}{4} \end{pmatrix}$$

$$(iv) Q^{-1}P^{-1} = \begin{pmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix} \begin{pmatrix} -1 & -\frac{1}{2} \\ -2 & -\frac{3}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -1 + 4 & -\frac{1}{2} + \frac{6}{2} \\ \frac{1}{2} - \frac{6}{2} & \frac{1}{4} - \frac{9}{4} \end{pmatrix}$$

$$= \begin{pmatrix} 3 & \frac{5}{2} \\ -\frac{5}{2} & -2 \end{pmatrix}$$

b). Berdasarkan hasil pada bagian a).

$$(i) (PQ)^{-1} = Q^{-1}P^{-1}$$

$$(ii) (QP)^{-1} = P^{-1}Q^{-1}$$

$$204) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow a + 3b = 1 \Leftrightarrow 2a + 6b = 2$$

$$\Rightarrow 2a + 4b = 0 \Leftrightarrow 2a + 4b = 0 -$$

$$2b = 2 \Leftrightarrow b = 1$$

$$\Rightarrow a = 1 - 3b = 1 - 3(1) = -2$$

$$\circ 2c + 4d = 1 \Leftrightarrow 2c + 4d = 1$$

$$\circ c + 3d = 0 \Leftrightarrow 2c + 6d = 0 -$$

$$-2d = 1$$

$$\Leftrightarrow d = -\frac{1}{2}$$

$$\Rightarrow c = -3d = \frac{3}{2}$$

\therefore Invers dari $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ adalah

$$\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

* Pemeriksaan

$$\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

b). misalkan $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ adalah matriks invers di masing-masing soal

$$(i) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow -3a + 2b = 1$$

$$\circ 2a - 2b = 0 +$$

$$-a = 1 \Leftrightarrow a = -1$$

$$\Rightarrow 2b = 2a \Leftrightarrow b = a = -1$$

$$\Leftrightarrow b = -1$$

$$\circ -3c + 2d = 0$$

$$2c - 2d = 1 +$$

$$-c = 1 \Leftrightarrow c = -1$$

$$\Rightarrow 2d = 2c - 1 = 2(-1) - 1 = -3$$

$$\Leftrightarrow d = -\frac{3}{2}$$

$$\therefore A = \begin{pmatrix} -1 & -1 \\ -1 & -\frac{3}{2} \end{pmatrix} \quad (25)$$

$$(ii) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow a \cdot 0 + b \cdot 1 = 1 \Leftrightarrow b = 1$$

$$\circ d = 0$$

$$\circ 2a + 4b = 0 \Leftrightarrow a = -2b = -2$$

$$\circ 2c + 4d = 1 \Leftrightarrow 2c = 1 - 4d = 1 -$$

$$\Leftrightarrow c = \frac{1}{2}$$

$$\therefore A = \begin{pmatrix} -2 & 1 \\ \frac{1}{2} & 0 \end{pmatrix}$$

$$(iii) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\circ a + 2b = 1$$

$$\circ a + 4b = 0 -$$

$$-2b = 1 \Leftrightarrow b = -\frac{1}{2}$$

$$\Rightarrow a = -4b = -4(-\frac{1}{2}) = 2 \Leftrightarrow a = 2$$

$$\circ c + 2d = 0$$

$$\circ c + 4d = 1 -$$

$$-2d = -1 \Leftrightarrow d = \frac{1}{2}$$

$$\Rightarrow c = -2d \Leftrightarrow c = -2(\frac{1}{2}) = -1$$

$$\therefore A = \begin{pmatrix} 2 & -\frac{1}{2} \\ -1 & \frac{1}{2} \end{pmatrix}$$

$$(iv) \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\circ 3a + 4b = 1 \Leftrightarrow 3a + 4b = 1$$

$$\circ a + 2b = 0 \Leftrightarrow 2a + 4b = 0 -$$

$$a = 1$$

$$\Rightarrow 2b = -a \Leftrightarrow b = -\frac{a}{2} = -\frac{1}{2} \Leftrightarrow b = -\frac{1}{2}$$

$$\circ 3c + 4d = 0 \Leftrightarrow 3c + 4d = 0$$

$$\circ c + 2d = 1 \Leftrightarrow 2c + 4d = 2 -$$

$$c = -2$$

$$\Rightarrow 4d = -3c \Leftrightarrow d = -\frac{3c}{4} = \frac{3}{2}$$

$$\therefore A = \begin{pmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{pmatrix}$$

Uji Pemahaman Halaman 73

21.a) Misalkan $R^{-1} = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\bullet aw + by = 1 \Leftrightarrow acw + bcy = c$$

$$\bullet cw + dy = 0 \Leftrightarrow acw + ady = 0$$

$$(bc - ad)y = c$$

$$\Leftrightarrow y = \frac{c}{bc - ad} = \frac{-c}{ad - bc}$$

$$\Rightarrow cw = -dy \Leftrightarrow w = -\frac{d}{c} \cdot y$$

$$\Leftrightarrow w = -\frac{d}{c} \cdot \frac{c}{bc - ad}$$

$$= -\frac{d}{bc - ad} = \frac{d}{ad - bc}$$

$$\bullet cx + dz = 1 \Leftrightarrow acx + adz = a$$

$$ax + bz = 0 \Leftrightarrow \frac{acx + bc z = 0}{(ad - bc)z = a}$$

$$\Leftrightarrow z = \frac{a}{ad - bc}$$

$$\Rightarrow ax = -bz \Leftrightarrow x = -\frac{b}{a} \cdot z$$

$$= -\frac{b}{a} \cdot \frac{a}{ad - bc}$$

$$= \frac{-b}{ad - bc}$$

$$\therefore R^{-1} = \begin{pmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{pmatrix}$$

$$b. R^2 \cdot (R^{-1})^2 = R \cdot R \cdot R^{-1} \cdot R^{-1}$$

$$= R \cdot (R \cdot R^{-1}) \cdot R^{-1}$$

$$= R \cdot I \cdot R^{-1}$$

$$= R \cdot R^{-1}$$

$$= I$$

\therefore Invers dari R^2 adalah $(R^{-1})^2$.

22.a) $\det(A) = 2 \cdot 3 - 1 \cdot 4 = 2$

$$A^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$$

$$\det(B) = 1 \cdot 3 - (-1)(-2) = 1$$

$$\Rightarrow B^{-1} = \frac{1}{1} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 5 \end{pmatrix}$$

$$\det(AB) = 0 \cdot 5 - 1 \cdot (-2) = 2$$

$$\bullet (AB)^{-1} = \frac{1}{2} \begin{pmatrix} 5 & -1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$$

$$\bullet B^{-1}A^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 1 & 0 \end{pmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

b). $BA = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 8 & 7 \end{pmatrix}$

$$\det(BA) = -2 \cdot 7 - (-2) \cdot 8 = 2$$

$$\Rightarrow (BA)^{-1} = \frac{1}{2} \begin{pmatrix} 7 & 2 \\ -8 & -2 \end{pmatrix} = \begin{pmatrix} \frac{7}{2} & 1 \\ -4 & -1 \end{pmatrix}$$

$$\Rightarrow \det((BA)^{-1}) = -\frac{7}{2} + 4 = \frac{1}{2}$$

$$\therefore |(BA)^{-1}| = \frac{1}{2}$$

23.a). $\det(A) = -3 \cdot 4 - 2 \cdot (-5) = -2$

$$\Rightarrow A^{-1} = \frac{1}{-2} \begin{pmatrix} 4 & -2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{5}{2} & \frac{3}{2} \end{pmatrix}$$

b). $\det(B) = 2 \cdot 1 - (-4)(-1) = -2$

$$\Rightarrow B^{-1} = \frac{1}{-2} \begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -2 \\ \frac{1}{2} & 1 \end{pmatrix}$$

c). $\det(C) = \cos^2 \theta - \sin \theta \cdot (-\sin \theta) = \cos^2 \theta + \sin^2 \theta = 1$

$$\Rightarrow C^{-1} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Jji Pemahaman Halaman 73

$$d). \det(D) = 4 \cdot 6 - 7 \cdot 9 = 1$$

$$\Rightarrow D^{-1} = \begin{pmatrix} 16 & -7 \\ -9 & 4 \end{pmatrix}$$

$$e). \det(E) = 1 \cdot 3 - 1 \cdot 3 = 0$$

$\therefore E^{-1}$ tidak ada

$$f). \det(F) = 1 \cdot 0 - 1 \cdot 1 = -1$$

$$\Rightarrow F^{-1} = \frac{1}{-1} \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$g). \det(G) = 2 \cdot (-3) - (-1) \cdot 6 = 0$$

$\therefore G^{-1}$ tidak ada

$$24. \det(M) = \frac{1}{2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right) \\ = \frac{1}{4} + \frac{3}{4}$$

$$\Rightarrow M^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$25. a). \det(M) = 9 \cdot 5 - 22 \cdot 2 = 1$$

$$\Rightarrow M^{-1} = \frac{1}{1} \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix} = \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix}$$

$$\begin{aligned} \circ M \cdot M^{-1} &= \begin{pmatrix} 9 & 22 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix} \\ &= \begin{pmatrix} 9 \cdot 5 + 22(-2) & 9 \cdot (-22) + 22 \cdot 9 \\ 2 \cdot 5 + 5(-2) & 2 \cdot (-22) + 5 \cdot 9 \end{pmatrix} \\ &= \begin{pmatrix} 45 - 44 & -198 + 198 \\ 10 - 10 & -44 + 45 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \circ M^{-1} \cdot M &= \begin{pmatrix} 5 & -22 \\ -2 & 9 \end{pmatrix} \begin{pmatrix} 9 & 22 \\ 2 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 5 \cdot 9 + (-22) \cdot 2 & 5 \cdot 22 + (-22) \cdot 5 \\ -2 \cdot 9 + 9 \cdot 2 & -2 \cdot 22 + 9 \cdot 5 \end{pmatrix} \\ &= \begin{pmatrix} 45 - 44 & 110 - 110 \\ -18 + 18 & -44 + 45 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

b). Berdasarkan Hasil dari bagian (a) dapat dilihat bahwa

$$M^{-1}M = MM^{-1} = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Jji Pemahaman Halaman 78

4. Pilihan Ganda

$$1. \begin{pmatrix} 2 & -5 \\ 4 & 7 \end{pmatrix} x = \begin{pmatrix} -12 & 1 \\ 10 & 19 \end{pmatrix}$$

$$x = \begin{pmatrix} 2 & -5 \\ 4 & 7 \end{pmatrix}^{-1} \begin{pmatrix} -12 & 1 \\ 10 & 19 \end{pmatrix}$$

$$= \frac{1}{14 - (-20)} \begin{pmatrix} 7 & 5 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} -12 & 1 \\ 10 & 19 \end{pmatrix}$$

$$= \frac{1}{34} \begin{pmatrix} -34 & 102 \\ 68 & 34 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 3 \\ 2 & 1 \end{pmatrix}$$

$$\Rightarrow \det(x) = (-1) \cdot 1 - 3 \cdot 2 = 1 - 6 = -5$$

Jawaban: C

$$2. x \begin{pmatrix} 6 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -6 & 8 \end{pmatrix}$$

$$x = \begin{pmatrix} 3 & -1 \\ -6 & 8 \end{pmatrix} \begin{pmatrix} 6 & 1 \\ -1 & 2 \end{pmatrix}^{-1}$$

$$x = \begin{pmatrix} 3 & -1 \\ -6 & 8 \end{pmatrix} \cdot \frac{1}{(6)(2) - (-1)(-6)} \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 \\ -6 & 8 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 3 \\ 4 & -6 \end{pmatrix}$$

$$\Rightarrow \det(x) = -5(-6) - 3(4) = 18$$

Jawaban: D

$$3. \begin{pmatrix} 6 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 20 & 32 \\ 10 & 2 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 1 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 20 & 32 \\ 10 & 2 \end{pmatrix}$$

$$= \frac{1}{6(-3) - 2(1)} \begin{pmatrix} -3 & -2 \\ -1 & 6 \end{pmatrix} \begin{pmatrix} 20 & 32 \\ 10 & 2 \end{pmatrix}$$

$$= \frac{1}{-20} \begin{pmatrix} -80 & -100 \\ 40 & -20 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ -2 & 1 \end{pmatrix}$$

$$\Rightarrow a+b+c+d = 4+5+(-2)+1 = 8$$

Jawaban: B.

$$4. \begin{pmatrix} h & 0 \\ k & 1 \end{pmatrix} \begin{pmatrix} 4 & -5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 8 & -10 \\ 5 & -2 \end{pmatrix}$$

$$\Rightarrow 4h = 8 \Leftrightarrow h = 2$$

$$\bullet 4k + 1 = 5 \Leftrightarrow 4k = 4 \Leftrightarrow k = 1$$

$$\therefore h+k = 2+1 = 3$$

Jawaban: C

$$5. \begin{pmatrix} 3 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$= \frac{1}{3 \cdot 3 - (-1) \cdot (-5)} \begin{pmatrix} 3 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$= \frac{1}{4} \begin{pmatrix} 20 \\ 36 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

$$\therefore p+q = 5+9 = 14$$

Jawaban: B

$$6. \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 7 & 9 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ -5 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 4 \\ 7 & 9 \end{pmatrix}$$

$$= \frac{1}{(-2) \cdot 3 - 1 \cdot (-5)} \begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 7 & 9 \end{pmatrix}$$

$$= \frac{1}{-1} \begin{pmatrix} -1 & 3 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 4 & -2 \end{pmatrix}$$

$$\therefore (d^2 - a^2)(c^2 - b^2)$$

$$= ((-2)^2 - 1^2)(4^2 - (-3)^2) = (4-1)(16-9)$$

$$= 3 \cdot 7 = 21$$

Jawaban: C

$$7. XA = B^T \Rightarrow X = B^T \cdot A^{-1}$$

$$= \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix}^T \cdot \begin{pmatrix} 3 & 5 \\ -1 & -2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} \cdot \frac{1}{-6 - (-5)} \begin{pmatrix} -2 & - \\ 1 & \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 13 \\ 13 & 34 \end{pmatrix}$$

Jawaban: B

Uji Pemahaman Halaman 78

$$8. A + X B = C D \Leftrightarrow X B = C D - A$$

$$\Leftrightarrow X = (C D - A) B^{-1}$$

$$\begin{aligned} \Rightarrow X &= \left(\begin{pmatrix} 1 & 3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} -9 & 2 \\ -1 & 8 \end{pmatrix} \right) \frac{1}{1} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \\ &= \left(\begin{pmatrix} 5 & 6 \\ 2 & 8 \end{pmatrix} - \begin{pmatrix} -9 & 2 \\ -1 & 8 \end{pmatrix} \right) \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 14 & 4 \\ 3 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 8 & -2 \\ 6 & -3 \end{pmatrix} \end{aligned}$$

Jawaban: A

$$9. K^{-1} X = L M \Leftrightarrow X = K L M$$

$$\begin{aligned} \Rightarrow X &= \begin{pmatrix} 5 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} -2 & 3 & 4 \\ 3 & -4 & -5 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ -3 & 1 \\ 4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 3 & 5 \\ 7 & -10 & -13 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ -3 & 1 \\ 4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 8 \\ -1 & -1 \end{pmatrix} \end{aligned}$$

$$\Rightarrow \det(X) = 9(-1) - 8(-8) = -9 + 64 = 55$$

Jawaban: A

$$10. A X = A^T A \Leftrightarrow X = A^{-1} A^T A$$

$$\begin{aligned} \Rightarrow X &= \frac{1}{16-24} \begin{pmatrix} 0 & -4 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \\ &= \frac{1}{-8} \begin{pmatrix} 0 & 16 \\ -4 & -20 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \\ &= \frac{1}{-8} \begin{pmatrix} 96 & 128 \\ -128 & -176 \end{pmatrix} \\ &= \frac{1}{-8} \begin{pmatrix} (-8)(-12) & (-8)(-16) \\ (-8)(16) & (-8)(22) \end{pmatrix} \\ &= \begin{pmatrix} -12 & -16 \\ 16 & 22 \end{pmatrix} \end{aligned}$$

Jawaban: D

B. Uraian

$$1. a). \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} Q = \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$$

$$\begin{aligned} Q &= \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}^{-1} \left(\begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} \right) \\ &= \frac{1}{1} \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 4 & 2 \end{pmatrix} \\ &= \begin{pmatrix} -19 & -11 \\ 14 & 8 \end{pmatrix} \end{aligned}$$

$$b). \begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 5 \end{pmatrix} = Q \begin{pmatrix} -3 & 2 \\ -1 & 2 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow Q &= \left(\begin{pmatrix} 5 & -3 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 5 \end{pmatrix} \right) \begin{pmatrix} -3 & 2 \\ -1 & 2 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 5 & -2 \\ -3 & 6 \end{pmatrix} \cdot \frac{1}{-4} \begin{pmatrix} 2 & -2 \\ 1 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 5 & -2 \\ -3 & 6 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{4} & \frac{3}{4} \end{pmatrix} \\ &= \begin{pmatrix} -2 & 1 \\ 0 & 3 \end{pmatrix} \end{aligned}$$

$$2. P = R - Q S \Leftrightarrow Q S = R - P$$

$$\Leftrightarrow Q = (R - P) S^{-1}$$

$$\begin{aligned} \Rightarrow Q &= \left(\begin{pmatrix} 0 & -1 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} -21 & -21 \\ -6 & -5 \end{pmatrix} \right) \begin{pmatrix} 3 & -2 \\ -5 & -8 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 21 & 20 \\ 8 & 6 \end{pmatrix} \cdot \frac{1}{-29-10} \begin{pmatrix} -8 & 2 \\ 5 & 3 \end{pmatrix} \\ &= \frac{1}{-39} \begin{pmatrix} 21 & 20 \\ 8 & 6 \end{pmatrix} \begin{pmatrix} -8 & 2 \\ 5 & 3 \end{pmatrix} \\ &= \frac{1}{-39} \begin{pmatrix} -68 & 102 \\ -34 & 39 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix} \end{aligned}$$

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3. a). $PR = Q \Leftrightarrow R = P^{-1}Q$

$$\Rightarrow R = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$$

$$= \frac{1}{1} \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -10 & -13 \\ 8 & 9 \end{pmatrix}$$

b). $RP = Q \Leftrightarrow R = Q P^{-1}$

$$\Rightarrow R = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix} \cdot \frac{1}{1} \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -17 \\ 10 & -13 \end{pmatrix}$$

4. $A \begin{pmatrix} 2 & 4 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 8 \\ 7 & 9 \end{pmatrix}$

$$\Leftrightarrow A = \begin{pmatrix} -3 & 8 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & -2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} -3 & 8 \\ 7 & 9 \end{pmatrix} \cdot \frac{1}{-4-4} \begin{pmatrix} -2 & -4 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{-8} \cdot \begin{pmatrix} -3 & 8 \\ 7 & 9 \end{pmatrix} \begin{pmatrix} -2 & -4 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{-8} \begin{pmatrix} -2 & 28 \\ -23 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} 0,25 & -3,5 \\ 2,875 & 1,25 \end{pmatrix}$$

5. $\begin{pmatrix} -6 & -5 \\ 4 & 4 \end{pmatrix} X = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

$$\Leftrightarrow X = \begin{pmatrix} -6 & -5 \\ 4 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \frac{1}{-24 - (-20)} \begin{pmatrix} 4 & 5 \\ -4 & -6 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \frac{1}{-4} \begin{pmatrix} 26 & 17 \\ -28 & -18 \end{pmatrix}$$

$$= \begin{pmatrix} -6,5 & -4,25 \\ 7 & 4,5 \end{pmatrix}$$

6. $X \begin{pmatrix} 2 & 7 \\ 5 & 3 \end{pmatrix} = \begin{pmatrix} -3 & 8 \\ 7 & -9 \end{pmatrix}$

$$X = \begin{pmatrix} -3 & 8 \\ 7 & -9 \end{pmatrix} \begin{pmatrix} 2 & 7 \\ 5 & 3 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} -3 & 8 \\ 7 & -9 \end{pmatrix} \cdot \frac{1}{6-35} \begin{pmatrix} 3 & -7 \\ -5 & 2 \end{pmatrix}$$

$$= \frac{1}{-29} \begin{pmatrix} -3 & 8 \\ 7 & -9 \end{pmatrix} \begin{pmatrix} 3 & -7 \\ -5 & 2 \end{pmatrix}$$

$$= \frac{1}{-29} \begin{pmatrix} -49 & 37 \\ 66 & -67 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{49}{29} & -\frac{37}{29} \\ -\frac{66}{29} & \frac{67}{29} \end{pmatrix}$$

7. $X \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{-4-6} \begin{pmatrix} -4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$= -\frac{1}{10} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$= -\frac{1}{10} \begin{pmatrix} -4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0,4 & 0,2 \\ 0,3 & -0,1 \end{pmatrix}$$

$\Rightarrow X \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0,4 & 0,2 \\ 0,3 & -0,1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0,5 \end{pmatrix}$

8. a) $MN = I \Leftrightarrow N = M^{-1}I = M^{-1}$

$$\Rightarrow N = \begin{pmatrix} 2 & -5 \\ 4 & -7 \end{pmatrix}^{-1} = \frac{1}{6} \begin{pmatrix} -7 & 5 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{7}{6} & \frac{5}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

b). $\begin{pmatrix} 2 & -5 \\ 4 & -7 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$a = -6$
 $b = -4$

$$\Leftrightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 4 & -7 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} -7 & 5 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ -4 \end{pmatrix}$$

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9. $PQ = I \Leftrightarrow P = I Q^{-1} = Q^{-1}$

$$\Rightarrow P = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}^{-1} = \frac{1}{-3} \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} & \frac{5}{3} \\ 1 & -\frac{4}{3} \end{pmatrix}$$

10. $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix}$

$$\begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

$$= \frac{1}{-3} \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 9 \\ 5 \end{pmatrix}$$

$$= \begin{pmatrix} 11 \\ -7 \end{pmatrix}$$

$$\Rightarrow m = 11; n = -7$$

11. $AB = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} 7 & 8 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ m & n \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\circ 3 \cdot 5 + 6m = 0 \Leftrightarrow 6m = -3 \cdot 5 \Leftrightarrow m = -2,5$$

$$\circ 5(-4) + 6n = 1 \Leftrightarrow 6n = 21 \Leftrightarrow n = 3,5$$

12. a). $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$x = 3$$

$$= \frac{1}{4-6} \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \Rightarrow y = -1$$

$$= \frac{1}{-2} \begin{pmatrix} -6 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

b). $\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 9 \\ 4 \end{pmatrix} \Rightarrow \begin{matrix} x = 5 \\ y = -2 \end{matrix}$$

$$= \frac{1}{7} \begin{pmatrix} 35 \\ -14 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

c). $\begin{pmatrix} -2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{-1} \begin{pmatrix} 2 & 1 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

$$\therefore x = 3, y = -5$$

d). $\begin{pmatrix} -3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 14 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -1 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 12 \\ 14 \end{pmatrix}$$

$$= \frac{1}{-10} \begin{pmatrix} 4 & -2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 12 \\ 14 \end{pmatrix}$$

$$= \frac{1}{-10} \begin{pmatrix} 20 \\ -30 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\therefore x = -2, y = 3$$

e). $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 8 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -7 \\ 8 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} -7 \\ 8 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\therefore x = -2; y = 3$$

13. $A^2 + mA + nI = 0$

$$\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} + m \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} + \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 8 \\ -4 & 7 \end{pmatrix} + \begin{pmatrix} m & 2m \\ -m & 3m \end{pmatrix} + \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow 8 + 2m + 0 = 0 \Leftrightarrow 2m = -8 \Leftrightarrow m = -4$$

$$\circ -1 + m + n = 0 \Leftrightarrow n = 1 - m = 1 - (-4) = 5$$

$$\therefore m = -4 \text{ dan } n = 5$$

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14. a) $A^{-1} = \frac{1}{-5} \begin{pmatrix} 3 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} -0,6 & 0,4 \\ 0,8 & -0,2 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 9 & 8 \\ 16 & 17 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 41 & 42 \\ 84 & 83 \end{pmatrix}$$

b) $f(A) = 2A^3 - 4A + 3I$

$$= 2 \begin{pmatrix} 41 & 42 \\ 84 & 83 \end{pmatrix} - 4 \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} + 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 82 & 84 \\ 168 & 166 \end{pmatrix} - \begin{pmatrix} 4 & 8 \\ 16 & 12 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 81 & 76 \\ 152 & 157 \end{pmatrix}$$

15. a) $A^2 = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 9 & -5 \\ -25 & 14 \end{pmatrix}$

$\Rightarrow f(A) = A^2 - 5A + I$

$$= \begin{pmatrix} 9 & -5 \\ -25 & 14 \end{pmatrix} - 5 \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -5 \\ -25 & 14 \end{pmatrix} - \begin{pmatrix} 10 & -5 \\ -25 & 15 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

b) $A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}^{-1} = \frac{1}{6-5} \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

$$= \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 9 & -5 \\ -25 & 14 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 43 & -24 \\ -120 & 67 \end{pmatrix}$$

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A. Pilihan Ganda

$$1. \begin{vmatrix} 1 & 2 & 3 \\ -2 & -1 & 3 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix}$$

$$= (1 \cdot (-1) \cdot 1 + 2 \cdot 3 \cdot 3 + 3 \cdot (-2) \cdot 2) - (3 \cdot (-1) \cdot 3 + 1 \cdot 3 \cdot 2 + 2 \cdot (-2) \cdot 1)$$

$$= (-1 + 18 + (-12)) - (-9 + 6 + (-4))$$

$$= 5 - (-7) = 12$$

Jawaban: E

2. Karena A matriks singular

$$\Leftrightarrow \det(A) = 0$$

$$\Leftrightarrow \begin{vmatrix} -1 & 1 & x-4 \\ 4 & x+2 & 4 \\ x-1 & 1 & 2x-4 \end{vmatrix} = 0$$

$$\Leftrightarrow (-1) \begin{vmatrix} x+2 & 4 \\ 1 & 2x-4 \end{vmatrix} - 1 \begin{vmatrix} 4 & 4 \\ x-1 & 2x-4 \end{vmatrix} + (x-4) \begin{vmatrix} 4 & x+2 \\ x-1 & 1 \end{vmatrix} = 0$$

$$\Leftrightarrow (-1) \cdot ((x+2)(2x-4) - 4 \cdot 1) - (4 \cdot (2x-4) - 4 \cdot (x-1)) + (x-4) (4 \cdot 1 - (x+2)(x-1)) = 0$$

$$\Leftrightarrow -(2x^2 - 8 - 4) - (4x - 12) + (x-4)(-x^2 - x + 6) = 0$$

$$+ (x-4)(-x^2 - x + 6) = 0$$

$$\Leftrightarrow (2x^2 - 8 - 4) + (4x - 12) + (x-4)(-x^2 - x + 6) = 0$$

$$+ (x-4)(-x^2 - x + 6) = 0$$

$$\Leftrightarrow 2x^2 + 4x - 24 + x^3 - 3x^2 - 10x + 24 = 0$$

$$\Leftrightarrow x^3 - x^2 - 6x = 0$$

$$\Leftrightarrow x(x^2 - x - 6) = 0 \Rightarrow x = 0, x = 3, \text{ atau } x = -2$$

$$\Leftrightarrow x(x-3)(x+2) = 0$$

Jawaban: C.

$$3. \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} - (-2) \begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix} + 3 \begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$= (2-3) + 2(-2-(-3)) + 3(-1-(-1))$$

$$= -1 + 2 = 1$$

$$\text{adj } A = \begin{pmatrix} \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} -2 & 3 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} -2 & 3 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -1 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} \end{pmatrix}$$

$$\text{adj } A = \begin{pmatrix} -1 & 7 & -9 \\ -1 & 5 & -6 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} -1 & 7 & -9 \\ -1 & 5 & -6 \\ 0 & 1 & -1 \end{pmatrix} = 0$$

$$0. |B| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 3 & 2 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 3 \end{vmatrix}$$

$$= (1 \cdot 1 \cdot 2 + 1 \cdot 1 \cdot 3 + 1 \cdot 2 \cdot 3) - (1 \cdot 1 \cdot 3 + 1 \cdot 1 \cdot 3 + 1 \cdot 2 \cdot 2)$$

$$= 11 - 10 = 1$$

$$\text{adj } B = \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$$

$$\text{adj } B = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 3 & 0 & -1 \end{pmatrix} \Rightarrow B^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

karena Adan 0 pasangan saling invers dan B tidak berpasangan
 \Rightarrow C juga tidak berpasangan

Jawaban: B.

$$4. DC = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -2 & 2 & -1 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -3 \\ -3 & 0 & -4 \\ -1 & -3 & -5 \end{pmatrix}$$

$$\Rightarrow \det(DC) = \begin{vmatrix} -1 & -2 & -3 \\ -3 & 0 & -4 \\ -1 & -3 & -5 \end{vmatrix}$$

$$= (0 + (-8) + (-27)) - (0 + (-30) + (-12))$$

$$= -35 + 42 = 7$$

$$\text{adj } DC = \begin{pmatrix} \begin{vmatrix} 0 & -4 \\ -3 & -5 \end{vmatrix} & -\begin{vmatrix} -2 & -3 \\ -3 & -5 \end{vmatrix} & \begin{vmatrix} -2 & -3 \\ 0 & -4 \end{vmatrix} \\ -\begin{vmatrix} -3 & -4 \\ -1 & -5 \end{vmatrix} & \begin{vmatrix} -1 & -3 \\ -1 & -5 \end{vmatrix} & -\begin{vmatrix} -1 & -3 \\ -3 & -4 \end{vmatrix} \\ \begin{vmatrix} -3 & 0 \\ -1 & -3 \end{vmatrix} & -\begin{vmatrix} -1 & -2 \\ -1 & -3 \end{vmatrix} & \begin{vmatrix} -1 & -2 \\ -3 & 0 \end{vmatrix} \end{pmatrix}$$

$$\text{adj } DC = \begin{pmatrix} -1 & -1 & 8 \\ -11 & 2 & 5 \\ 9 & -1 & -6 \end{pmatrix}$$

$$\Rightarrow DC^{-1} = \frac{1}{7} \begin{pmatrix} -1 & -1 & 8 \\ -11 & 2 & 5 \\ 9 & -1 & -6 \end{pmatrix}$$

Jawaban: A.

$$5. AB = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 & 1 \\ 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\det AB = (-3 + 0 + 6) - (0 + 6 + 6) = 3 - 12 = -9$$

$$\text{adj } AB = \begin{pmatrix} \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 3 & -1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 0 & 3 \\ -3 & 3 & 0 \\ 6 & -6 & -9 \end{pmatrix}$$

$$\Rightarrow AB^{-1} = \frac{1}{-9} \begin{pmatrix} -3 & 0 & 3 \\ -3 & 3 & 0 \\ 6 & -6 & -9 \end{pmatrix}$$

$$= \frac{1}{3} \cdot \left(\frac{1}{3} \begin{pmatrix} -(-3) & 0 & -3 \\ -(-3) & -3 & 0 \\ -6 & -(-6) & -(-9) \end{pmatrix} \right)$$

$$= \frac{1}{3} \left(\frac{1}{3} \begin{pmatrix} 3 & 0 & -3 \\ 3 & -3 & 0 \\ -6 & 6 & 9 \end{pmatrix} \right)$$

$$= \frac{1}{3} C$$

$$\therefore C = \frac{1}{3} \begin{pmatrix} 3 & 0 & -3 \\ 3 & -3 & 0 \\ -6 & 6 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ -2 & 2 & 3 \end{pmatrix}$$

Jawaban: B.

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B. Uraian

$$1. a). \begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & -1 \\ -2 & -1 \end{vmatrix}$$

$$= (1 \cdot (-1) \cdot 3 + 2 \cdot 4 \cdot (-2) + 0 \cdot 3 \cdot 1)$$

$$- (0 \cdot (-1) \cdot (-2) + 1 \cdot 4 \cdot 1 + 3 \cdot 2 \cdot 3)$$

$$= (-3 - 16 + 0) - (0 + 4 + 18)$$

$$= -19 - 22$$

$$= -41$$

$$b). \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= -0 \cdot \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 0 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 0 + 1 \cdot (2 - (-1)) + 0$$

$$= 3$$

$$c). \begin{vmatrix} 1 & 3 & 1 \\ -1 & 2 & 1 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= 0 \cdot \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix}$$

$$= 0 - 2 \cdot (1 - (-1)) + 0$$

$$= -4$$

$$d). \begin{vmatrix} 2 & -1 & 0 \\ -3 & 1 & 2 \\ 1 & -3 & 1 \end{vmatrix} \begin{vmatrix} 2 & -1 \\ -3 & 1 \\ 1 & -3 \end{vmatrix}$$

$$= (2 \cdot 1 \cdot 1 + (-1) \cdot 2 \cdot 1 + 0 \cdot (-3) \cdot (-3))$$

$$- (0 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot (-3) + (-1) \cdot (-3) \cdot 1)$$

$$= (2 - 2 + 0) - (0 - 12 + 3)$$

$$= 9$$

$$e). \begin{vmatrix} 2 & 4 & -1 \\ -1 & 3 & 2 \\ 4 & 0 & 2 \end{vmatrix} \begin{vmatrix} 2 & 4 \\ -1 & 3 \\ 4 & 0 \end{vmatrix}$$

$$= (2 \cdot 3 \cdot 2 + 4 \cdot 2 \cdot 4 + (-1) \cdot (-1) \cdot 0) - ((-1) \cdot 3 \cdot 4 + 2 \cdot 2 \cdot 0 + 4 \cdot (-1) \cdot 2)$$

$$= (12 + 32 + 0) - (-12 + 0 - 8)$$

$$= 44 + 20$$

$$= 64$$

$$f). \begin{vmatrix} 2 & 1 & 4 \\ 3 & 2 & 6 \\ 5 & -3 & 10 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 3 & 2 \\ 5 & -3 \end{vmatrix}$$

$$= (2 \cdot 2 \cdot 10 + 1 \cdot 6 \cdot 5 + 4 \cdot 3 \cdot (-3)) - (4 \cdot 2 \cdot 5 + 2 \cdot 6 \cdot (-3) + 1 \cdot 3 \cdot 10)$$

$$= (40 + 30 - 36) - (40 - 36 + 30)$$

$$= 0$$

$$g). \begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 2 & 0 & 2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 0 \end{vmatrix}$$

$$= (1 \cdot (-1) \cdot 2 + 2 \cdot 2 \cdot 2 + 3 \cdot 3 \cdot 0) - (3 \cdot (-1) \cdot 2 + 1 \cdot 2 \cdot 0 + 2 \cdot 3 \cdot 2)$$

$$= (-2 + 8 + 0) - (-6 + 0 + 12)$$

$$= 6 - 6$$

$$= 0$$

$$= 0$$

$$h). \begin{vmatrix} 6 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix} = 6 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 6 \cdot 4 = 24$$

$$i). \begin{vmatrix} -1 & 0 & 2 \\ -2 & 1 & 0 \\ 2 & 1 & -3 \end{vmatrix} \begin{vmatrix} -1 & 0 \\ -2 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= ((-1) \cdot 1 \cdot (-3) + 0 \cdot 0 \cdot 2 + 2 \cdot (-2) \cdot 1) - (2 \cdot 1 \cdot 2 + (-1) \cdot 0 \cdot 1 + 0 \cdot (-2) \cdot (-3))$$

$$= (3 + 0 - 4) - (4 + 0 + 0)$$

$$= -5$$

$$d). \begin{vmatrix} 2 & 5 & -1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} = -5 \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = -5(1) = -5$$

$$2.a). \begin{vmatrix} x & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 1 & 4 \end{vmatrix} = 3$$

$$\Leftrightarrow x \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 3 \Leftrightarrow x(4-3) = 3$$

$$\Leftrightarrow x = 3.$$

$$b). \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$$

$$\Leftrightarrow (x^2 \cdot 2 \cdot 4 + x \cdot 1 \cdot 3 + 1 \cdot 0 \cdot 1) - (1 \cdot 2 \cdot 3 + x^2 \cdot 1 \cdot 1 + x \cdot 0 \cdot 4) = 28$$

$$\Leftrightarrow (8x^2 + 3x + 0) - (6 + x^2 + 0) = 28$$

$$\Leftrightarrow 7x^2 + 3x - 6 = 28$$

$$\Leftrightarrow 7x^2 + 3x - 34 = 0$$

$$\Leftrightarrow 7x^2 + 17x - 19x - 34 = 0$$

$$\Leftrightarrow x(7x + 17) - 2(7x + 17) = 0$$

$$\Leftrightarrow (x - 2)(7x + 17) = 0$$

$$\Leftrightarrow x = 2 \text{ atau } x = -\frac{17}{7}$$

$$c). \begin{vmatrix} x^2 & 0 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & 0 \end{vmatrix} = 1$$

$$\Leftrightarrow (x^2 \cdot (-1) \cdot 0 + 0 \cdot 3 \cdot 3 + 1 \cdot 2 \cdot 2) - (1 \cdot (-1) \cdot 3 + 3 \cdot 2 \cdot x^2 + 0 \cdot 2 \cdot 0) = 1$$

$$\Leftrightarrow 4 - (6x^2 - 3) = 1$$

$$\Leftrightarrow -6x^2 + 7 = 1$$

$$\Leftrightarrow 6x^2 = 6$$

$$\Leftrightarrow x^2 = 1$$

$$\Leftrightarrow x = 1 \text{ atau } x = -1$$

$$d). \begin{vmatrix} x & 1 & 1 \\ 0 & x & 1 \\ 0 & x & 0 \end{vmatrix} = -4$$

$$\Leftrightarrow x \begin{vmatrix} x & 1 \\ x & 0 \end{vmatrix} = -4$$

$$\Leftrightarrow x(-x) = -4$$

$$\Leftrightarrow x^2 = 4$$

$$\Leftrightarrow x = 2 \text{ atau } x = -2.$$

$$3.a). |A| = \begin{vmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= (2 + 2 + 0) - (3 + 8 + 0)$$

$$= -7$$

$$|A^T| = \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= (2 + 0 + 2) - (3 + 8 + 0)$$

$$= -7$$

$$\therefore |A| = |A^T|$$

$$b). |A| = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= (0 + 4 - 3) - (0 + 2 + 12) = -13$$

$$|A^T| = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 0 & 1 \\ -1 & 2 & 2 \end{vmatrix}$$

$$= (0 - 3 + 4) - (0 + 2 + 12) = -13$$

$$\therefore |A| = |A^T|$$

$$c). |A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 2 & 0 & 2 \end{vmatrix}$$

$$= (-2 + 8 + 0) - (-6 + 0 + 12) = 0$$

$$|A^T| = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & 0 \\ 3 & 2 & 2 \end{vmatrix}$$

$$= (-2 + 0 + 8) - (-6 + 0 + 12) = 0$$

$$\therefore |A| = |A^T|$$

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4. a) Misalkan

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 3 & 1 & 4 \\ 5 & 2 & 6 \\ 8 & 7 & 9 \end{pmatrix}$$

$$\begin{aligned} \circ \det A &= (45 + 126 + 64) - (140 + 98 + 54) \\ &= 235 - 242 \\ &= -7. \end{aligned}$$

$$\begin{aligned} \circ |B| &= (54 + 98 + 140) - (64 + 126 + 95) \\ &= 242 - 235 \\ &= 7 \end{aligned}$$

$$\therefore \text{ terbukti } |A| = -|B|$$

b). Misalkan

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 1 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\circ |A| = (5 + 0 + 0) - (0 + 0 + 8) = -3$$

$$\circ |B| = (0 + 0 + 0) - (0 + 0 + 5) = -5$$

$$\therefore \text{ Terbukti } |A| = -|B|$$

c). Misalkan

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 7 & 8 & 9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}$$

$$\begin{aligned} \circ |A| &= (45 + 126 + 64) - (140 + 98 + 54) \\ &= -7 \end{aligned}$$

$$\begin{aligned} \circ |B| &= (140 + 98 + 54) - (45 + 126 + 64) \\ &= 7 \end{aligned}$$

$$\therefore \text{ Terbukti } |A| = -|B|$$

$$5. a). \begin{vmatrix} 1 & 1 & 8 \\ 3 & 5 & 0 \\ 2 & 7 & 0 \end{vmatrix} = 0 \begin{vmatrix} 3 & 5 \\ 2 & 7 \end{vmatrix} - 0 \begin{vmatrix} 1 & 1 \\ 2 & 7 \end{vmatrix} + 0 \begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} = 0$$

$$b). \begin{vmatrix} 3 & -2 & 4 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{vmatrix} = (0 + 0 + 0) - (0 + 0 + 0) = 0$$

$$c). \begin{vmatrix} 7 & 0 & -9 \\ -1 & 0 & 6 \\ 2 & 0 & 5 \end{vmatrix} = (0 + 0 + 0) - (0 + 0 + 0) = 0$$

6. a). Misalkan

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 2 & 6 & 8 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

$$|A| = (8 + 0 + 0) - (0 + 18 + 0) = -10$$

$$|B| = (4 + 0 + 0) - (0 + 9 + 0) = -5$$

$$\Rightarrow |A| = -10 = 2(-5) = 2|B|$$

$$\therefore |A| = 2|B| \text{ (terbukti)}$$

b). Misalkan

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 1 & -2 \\ 6 & 1 & 1 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix}$$

$$|A| = (2 + 0 + 4) - (6 - 4 + 0) = 4$$

$$|B| = (1 + 0 + 2) - (3 - 2 + 0) = 2$$

$$\Rightarrow |A| = 4 = 2 \cdot 2 = 2|B|$$

$$\therefore |A| = 2|B| \text{ (terbukti)}$$

c). Misalkan

$$A = \begin{pmatrix} 4 & 5 & 8 \\ -1 & 1 & 2 \\ 3 & 1 & 6 \end{pmatrix}, \text{ dan } B = \begin{pmatrix} 4 & 5 & 4 \\ -1 & 1 & 1 \\ 3 & 1 & 3 \end{pmatrix}$$

$$|A| = (24 + 30 - 8) - (24 + 8 - 30) = 44$$

$$|B| = (12 + 15 - 4) - (12 + 4 - 15) = 22$$

$$\Rightarrow |A| = 44 = 2 \cdot 22 = 2|B|$$

$$\therefore |A| = 2|B|$$

$$7. a). \begin{vmatrix} 3 & 1 & -5 \\ 2 & 0 & 4 \\ -2 & 0 & -1 \end{vmatrix} = (0 - 8 + 0) - (0 + 0 - 2) = -6$$

$$b). \begin{vmatrix} -2 & 3 & 1 \\ -1 & 0 & 0 \\ 4 & -1 & 5 \end{vmatrix} = (0 + 0 + 1) - (0 + 0 - 15) = 16$$

$$8. a). \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix}$$

$$= (8 + 27 + 1) - (6 + 6 + 6)$$

$$= 18$$

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$$\text{adj } A = \begin{pmatrix} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} \\ -\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{18} \begin{pmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{pmatrix}$$

b). $|B| = \begin{vmatrix} 2 & 1 & 1 \\ -2 & 3 & 0 \\ 3 & 5 & 2 \end{vmatrix}$

$$= (12 + 0 - 10) - (9 + 0 - 4)$$

$$= -3$$

$$\text{adj } B = \begin{pmatrix} \begin{vmatrix} 3 & 0 \\ 5 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 5 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix} \\ -\begin{vmatrix} -2 & 0 \\ 3 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ -2 & 0 \end{vmatrix} \\ \begin{vmatrix} -2 & 3 \\ 3 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ -2 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 3 & -3 \\ 4 & 1 & -2 \\ -19 & -7 & 0 \end{pmatrix}$$

$$\Rightarrow B^{-1} = -\frac{1}{3} \begin{pmatrix} 6 & 3 & -3 \\ 4 & 1 & -2 \\ -19 & -7 & 0 \end{pmatrix}$$

c). $|C| = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 6 \\ 1 & 2 & 1 \end{vmatrix}$

$$= (1 + 18 + 0) - (2 + 12 + 6)$$

$$= 27 - 20$$

$$= 7$$

$$\text{adj } C = \begin{pmatrix} \begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} \\ -\begin{vmatrix} 2 & 6 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -11 & 1 & 16 \\ 4 & -1 & -2 \\ 3 & 1 & -5 \end{pmatrix}$$

$$\Rightarrow C^{-1} = \frac{1}{7} \begin{pmatrix} -11 & 1 & 16 \\ 4 & -1 & -2 \\ 3 & 1 & -5 \end{pmatrix}$$

d). $|D| = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 3 & 4 \\ 2 & 1 & 1 \end{vmatrix}$

$$= (3 + 24 + 2) - (12 + 4 + 3)$$

$$= 29 - 19$$

$$= 10$$

$$\text{adj } D = \begin{pmatrix} \begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -1 & 6 \\ 7 & -3 & -2 \\ -5 & 5 & 0 \end{pmatrix}$$

$$\Rightarrow D^{-1} = \frac{1}{10} \begin{pmatrix} -1 & -1 & 6 \\ 7 & -3 & -2 \\ -5 & 5 & 0 \end{pmatrix}$$

$$e). |E| = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 7 \\ 1 & -6 & -2 \end{vmatrix}$$

$$= (-2 + 21 - 6) - (1 - 42 - 6)$$

$$= 13 - (-47)$$

$$= 60$$

$$\text{adj } E = \begin{pmatrix} \begin{vmatrix} 1 & 7 \\ -6 & -2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix} \\ -\begin{vmatrix} 1 & 7 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 7 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & -6 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & -6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 40 & 0 & 20 \\ 9 & -3 & -6 \\ -7 & 9 & -2 \end{pmatrix}$$

$$\Rightarrow E^{-1} = \frac{1}{60} \begin{pmatrix} 40 & 0 & 20 \\ 9 & -3 & -6 \\ -7 & 9 & -2 \end{pmatrix}$$

$$f). |P| = \begin{vmatrix} 2 & 4 & 3 \\ 3 & 6 & 5 \\ 2 & 5 & 2 \end{vmatrix}$$

$$= (24 + 40 + 45) - (36 + 50 + 24)$$

$$= 109 - 110$$

$$= -1$$

$$\text{adj } P = \begin{pmatrix} \begin{vmatrix} 6 & 5 \\ 5 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ 5 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 2 & 5 \end{vmatrix} \\ -\begin{vmatrix} 3 & 5 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} \\ \begin{vmatrix} 3 & 6 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 4 \\ 2 & 5 \end{vmatrix} & \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} \end{pmatrix}$$

$$\text{adj } P = \begin{pmatrix} -13 & 7 & 2 \\ 4 & -2 & -1 \\ 3 & -2 & 0 \end{pmatrix}$$

$$\Rightarrow P^{-1} = -\begin{pmatrix} -13 & 7 & 2 \\ 4 & -2 & -1 \\ 3 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 13 & -7 & -2 \\ -4 & 2 & 1 \\ -3 & 2 & 0 \end{pmatrix}$$

$$g). |Q| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 3 \end{vmatrix}$$

$$= (9 + 12 + 12) - (18 + 12 + 6)$$

$$= 33 - 36$$

$$= -3$$

$$\text{adj } Q = \begin{pmatrix} \begin{vmatrix} 3 & 3 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} \\ -\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 6 & -3 \\ 3 & -3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow Q^{-1} = -\frac{1}{3} \begin{pmatrix} -3 & 6 & -3 \\ 3 & -3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$h). |R| = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= (6 - 2 - 2) - (3 + 8 + 1)$$

$$= -10$$

$$\text{adj } R = \begin{pmatrix} -1 & -3 & 5 \\ -3 & 1 & -5 \\ 5 & -5 & 5 \end{pmatrix}$$

$$\therefore R^{-1} = -\frac{1}{10} \begin{pmatrix} -1 & -3 & 5 \\ -3 & 1 & -5 \\ 5 & -5 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 0,1 & 0,3 & -0,5 \\ 0,3 & -0,1 & 0,5 \\ -0,5 & 0,5 & -0,5 \end{pmatrix}$$

Jji Pemahaman Halaman 92

3.a).
(i) $AB = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix} \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$
 $= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(ii) $|AB| = 3 \cdot 3 \cdot 3 = 27$

adj AB

$$= \begin{pmatrix} 3 \cdot 3 - 0 \cdot 0 & -(0 \cdot 3 - 0 \cdot 0) & 0 \cdot 0 - 0 \cdot 3 \\ -(0 \cdot 3 - 0 \cdot 0) & 3 \cdot 3 - 0 \cdot 0 & -(3 \cdot 0 - 0 \cdot 0) \\ 0 \cdot 0 - 3 \cdot 0 & -(3 \cdot 0 - 0 \cdot 0) & 3 \cdot 3 - 0 \cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$\Rightarrow (AB)^{-1} = \frac{1}{27} \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

(iii) $|A| = (10 + 8 + 30) - (9 + 20 + 24) = 3$

adj A

$$= \begin{pmatrix} 3 \cdot 6 - 4 \cdot 5 & -(2 \cdot 6 - 3 \cdot 5) & 2 \cdot 4 - 3 \cdot 3 \\ -(2 \cdot 6 - 4 \cdot 1) & 1 \cdot 6 - 3 \cdot 1 & -(1 \cdot 4 - 3 \cdot 2) \\ 2 \cdot 5 - 3 \cdot 1 & -(1 \cdot 5 - 2 \cdot 1) & 1 \cdot 3 - 2 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{3} \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$$

(iv) $|B| = (6 + 92 - 24) - (-2 + 12 + 24) = 9$

adj B

$$= \begin{pmatrix} 3 & 6 & 9 \\ 6 & 9 & 12 \\ 3 & 15 & 18 \end{pmatrix}$$

$$\Rightarrow B^{-1} = \frac{1}{9} \begin{pmatrix} 3 & 6 & 9 \\ 6 & 9 & 12 \\ 3 & 15 & 18 \end{pmatrix}$$

$$B^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix}$$

(v) $B^{-1}A^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix} \cdot \frac{1}{3} \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$

$$= \frac{1}{9} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

b) Berdasarkan hasil pada bagian (a, v) dan (a, ii) maka

$$(AB)^{-1} = B^{-1}A^{-1}$$

10. $PQ = \begin{pmatrix} 5 & 0 & 0 \\ 6 & 14 & 0 \\ 6 & 3 & 11 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ -2 & -4 & 0 \\ -2 & -1 & -3 \end{pmatrix}$

$$= \begin{pmatrix} -5 & 0 & 0 \\ -34 & -56 & 0 \\ -34 & -23 & -33 \end{pmatrix}$$

• $QP = \begin{pmatrix} -1 & 0 & 0 \\ -2 & -4 & 0 \\ -2 & -1 & -3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 0 \\ 6 & 14 & 0 \\ 6 & 3 & 11 \end{pmatrix}$

$$= \begin{pmatrix} -5 & 0 & 0 \\ -34 & -56 & 0 \\ -34 & -23 & -33 \end{pmatrix}$$

Karena $PQ = QP$ dan

$$|PQ| = (-5)(-56)(-33) \neq 0$$

$$\Rightarrow (PQ)^{-1} = (QP)^{-1} //$$

Ji Pemahaman Halaman 105

t. Pilihan Ganda

$$1. \begin{cases} 9x - 5y = 19 \\ 7x - 4y = 15 \end{cases} \Rightarrow \underbrace{\begin{pmatrix} 9 & -5 \\ 7 & -4 \end{pmatrix}}_A \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_t = \underbrace{\begin{pmatrix} 19 \\ 15 \end{pmatrix}}_B$$

$$\circ A t = B \Leftrightarrow t = A^{-1} B$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 & -5 \\ 7 & -4 \end{pmatrix}^{-1} \begin{pmatrix} 19 \\ 15 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 7 & -9 \end{pmatrix} \begin{pmatrix} 19 \\ 15 \end{pmatrix}$$

Jawaban: E

$$2. \begin{cases} 5x + 13y = 13 \\ 3x + 5y = 19 \end{cases} \Rightarrow \begin{pmatrix} 5 & 13 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 19 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ 3 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 13 \\ 19 \end{pmatrix}$$

$$= \frac{1}{25-39} \begin{pmatrix} 5 & -13 \\ -3 & 5 \end{pmatrix} \begin{pmatrix} 13 \\ 19 \end{pmatrix}$$

$$= \frac{1}{-14} \begin{pmatrix} 5(13) + 19(-13) \\ -3(13) + 5(19) \end{pmatrix}$$

$$= \frac{1}{-14} \begin{pmatrix} -182 \\ 56 \end{pmatrix}$$

$$= \begin{pmatrix} 13 \\ -4 \end{pmatrix}$$

$$\therefore x + y = 13 + (-4) = 9$$

Jawaban: B.

$$3. \begin{cases} 9x + 4y = 29 \\ 15x - 8y = -3 \end{cases} \Rightarrow \begin{pmatrix} 9 & 4 \\ 15 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 29 \\ -3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 & 4 \\ 15 & -8 \end{pmatrix}^{-1} \begin{pmatrix} 29 \\ -3 \end{pmatrix}$$

$$= \frac{1}{-72-60} \begin{pmatrix} -8 & -4 \\ -15 & 9 \end{pmatrix} \begin{pmatrix} 29 \\ -3 \end{pmatrix}$$

$$= \frac{1}{-132} \begin{pmatrix} -8 \cdot 29 + (-4)(-3) \\ -15 \cdot 29 + 9(-3) \end{pmatrix}$$

$$= -\frac{1}{132} \begin{pmatrix} -220 \\ -462 \end{pmatrix}$$

$$= \frac{1}{12} \begin{pmatrix} 20 \\ 42 \end{pmatrix}$$

$$\Rightarrow 12(x+y) = 12 \cdot \frac{1}{12} (20+42) = 62 \quad (41)$$

Jawaban: E.

$$4. \begin{cases} x+y-z = 29 \\ 2x-y+2z = 4 \\ x+2y-3z = 36 \end{cases} \Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 29 \\ 4 \\ 36 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 2 \\ 1 & 2 & -3 \end{vmatrix}$$

$$= (3+2-4) - (1+4-6)$$

$$= 2.$$

$$\circ D_x = \begin{vmatrix} 29 & 1 & -1 \\ 4 & -1 & 2 \\ 36 & 2 & -3 \end{vmatrix}$$

$$= (72+72-8) - (-36+96-12)$$

$$= 136 - 120$$

$$= 16$$

$$\circ D_y = \begin{vmatrix} 1 & 29 & -1 \\ 2 & 4 & 2 \\ 1 & 36 & -3 \end{vmatrix}$$

$$= (-12+48-72) - (-4+72-144)$$

$$= -36 - (-76)$$

$$= 40$$

$$\circ D_z = \begin{vmatrix} 1 & 1 & 29 \\ 2 & -1 & 4 \\ 1 & 2 & 36 \end{vmatrix}$$

$$= (-36+4+96) - (-29+0+72)$$

$$= 64 - 56$$

$$= 8$$

$$\Rightarrow x:y:z = \frac{16}{2} : \frac{40}{2} : \frac{8}{2} = 8:20:4$$

$$= 8:20:4$$

$$= 2:5:1$$

Jawaban: C.

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$$\begin{aligned} 5. 2x - 4y + 3z &= 8 \\ x + 2y - z &= 5 \\ -2x - 3y + z &= -9 \end{aligned} \Rightarrow \begin{pmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$$

$$D = \begin{vmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{vmatrix}$$

$$= (4 - 8 - 9) - (-12 + 6 - 4)$$

$$= -13 - (-10)$$

$$= -3$$

$$D_x = \begin{vmatrix} 8 & -4 & 3 \\ 5 & 2 & -1 \\ -9 & -3 & 1 \end{vmatrix}$$

$$= (16 - 36 - 45) - (-54 + 29 - 20)$$

$$= -65 + 50$$

$$= -15$$

$$D_y = \begin{vmatrix} 2 & 8 & 3 \\ 1 & 5 & -1 \\ -2 & -9 & 1 \end{vmatrix}$$

$$= (10 + 16 - 27) - (-30 + 18 + 8)$$

$$= -1 - (-4)$$

$$= 3$$

$$D_z = \begin{vmatrix} 2 & -4 & 8 \\ 1 & 2 & 5 \\ -2 & -3 & -9 \end{vmatrix}$$

$$= (-36 + 40 - 24) - (-32 - 30 + 36)$$

$$= -20 - (-26)$$

$$= 6$$

$$\Rightarrow x + y + z = \frac{D_x}{D} + \frac{D_y}{D} + \frac{D_z}{D}$$

$$= \frac{-15}{-3} + \frac{3}{-3} + \frac{6}{-3}$$

$$= 5 - 1 - 2$$

$$= 2$$

Jawaban: A.

B. Uraian

$$1. a). \begin{aligned} 4x + 9y &= 3 \\ 3x + 5y &= 8 \end{aligned} \Rightarrow \begin{pmatrix} 4 & 9 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$$

$$b). \begin{aligned} 3x - 5y &= 15 \\ 2x + y &= -4 \end{aligned} \Rightarrow \begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ -4 \end{pmatrix}$$

$$c). \begin{aligned} x - 2y &= 5 \\ 4x + y &= 7 \end{aligned} \Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$d). \begin{aligned} -2x + 7y &= 1 \\ 4x - 3y &= 6 \end{aligned} \Rightarrow \begin{pmatrix} -2 & 7 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$$

$$e). \begin{aligned} 9x - 2y &= -6 \\ -x - 3y &= 11 \end{aligned} \Rightarrow \begin{pmatrix} 9 & -2 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 11 \end{pmatrix}$$

$$2. \begin{pmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2x - 4y + 3z = -2 \\ x + 2y - z = 1 \\ -2x - 3y + z = 4 \end{cases}$$

$$3. a). \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= \frac{1}{4-6} \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$= -\frac{1}{2} \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$\therefore x = 3, \text{ dan } y = -1$$

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$$b). \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 4 \end{pmatrix} \\ &= \frac{1}{3 - (-4)} \begin{pmatrix} 3 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 9 \\ 4 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 35 \\ -14 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ -2 \end{pmatrix} \end{aligned}$$

$$\therefore x = 5; y = -2.$$

$$c). \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{-4 - 3} \begin{pmatrix} 2 & -1 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix} \\ &= -\frac{1}{7} \begin{pmatrix} -1 \\ 5 \end{pmatrix} \end{aligned}$$

$$\therefore x = \frac{1}{7} \text{ dan } y = -\frac{5}{7}.$$

$$d). \begin{pmatrix} -3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 14 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{-12 - (-2)} \begin{pmatrix} 4 & -2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 12 \\ 14 \end{pmatrix} \\ &= -\frac{1}{10} \begin{pmatrix} 20 \\ -30 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \end{aligned}$$

$$\therefore x = -2 \text{ dan } y = 3.$$

$$e). \begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \frac{1}{-4 - (-6)} \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} -7 \\ 0 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ 3 \end{pmatrix} \end{aligned}$$

$$\therefore x = -2 \text{ dan } y = 3.$$

$$4. a). \begin{pmatrix} 7 & 2 \\ -2 & 0 \end{pmatrix}^{-1} \frac{1}{60} \begin{pmatrix} 8 & -2 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} \frac{2}{15} & -\frac{1}{30} \\ \frac{1}{30} & \frac{7}{60} \end{pmatrix}$$

$$b). 7x + 2y = 12$$

$$-2x + 0y = -12$$

$$\Rightarrow \begin{pmatrix} 7 & 2 \\ -2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -12 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 7 & 2 \\ -2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 12 \\ -12 \end{pmatrix} \\ &= \frac{1}{60} \begin{pmatrix} 8 & -2 \\ 2 & -7 \end{pmatrix} \begin{pmatrix} 12 \\ -12 \end{pmatrix} \end{aligned}$$

$$= \frac{1}{60} \begin{pmatrix} 120 \\ -60 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\therefore x = 2 \text{ dan } y = -1$$

$$5. \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}^{-1} = \frac{1}{6 - (-1)} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$a). \begin{aligned} 2x - y &= -11 \\ x + 3y &= 12 \end{aligned} \Rightarrow \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -11 \\ 12 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} -11 \\ 12 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -11 \\ 12 \end{pmatrix} \end{aligned}$$

$$= \frac{1}{7} \begin{pmatrix} -21 \\ 35 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

$$\therefore x = -3, y = 5$$

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$$\begin{aligned} \text{b). } 2x - y &= 9 \\ x + 3y &= 1 \end{aligned} \Rightarrow \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 1 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 1 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 9 \\ 1 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 28 \\ -7 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ -1 \end{pmatrix} \end{aligned}$$

$$\therefore x = 4; y = -1$$

$$\begin{aligned} \text{c). } 4x - 2y - 5 &= 0 \Leftrightarrow 2x - y = \frac{5}{2} \\ 2x + 6y + 1 &= 0 \Leftrightarrow x + 3y = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} \frac{5}{2} \\ -\frac{1}{2} \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 7 \\ -\frac{7}{2} \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} \end{aligned}$$

$$\therefore x = 1; y = -\frac{1}{2}$$

$$\begin{aligned} \text{6.a). } x - y &= 1 \\ x + y &= 5 \end{aligned} \Rightarrow \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\begin{aligned} \Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \\ &= \frac{1}{1 - (-1)} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 6 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \end{aligned}$$

$$\therefore \text{Himpunan penyelesaian} = \{(3, 2)\}$$

$$\begin{aligned} \text{b). } 2x - 3y &= 6 \\ x + 3y &= 3 \end{aligned} \Rightarrow \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -3 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \\ &= \frac{1}{6 - (-3)} \begin{pmatrix} 3 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \end{pmatrix} \\ &= \frac{1}{9} \begin{pmatrix} 27 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 0 \end{pmatrix} \end{aligned}$$

$$\therefore \text{Himpunan penyelesaian} = \{(3, 0)\}$$

$$\begin{aligned} \text{c). } 3x + y &= 7 \\ 2x - 5y &= -1 \end{aligned} \Rightarrow \begin{pmatrix} 3 & 1 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 3 & 1 \\ 2 & -5 \end{pmatrix}^{-1} \begin{pmatrix} 7 \\ -1 \end{pmatrix} \\ &= \frac{1}{-15 - 2} \begin{pmatrix} -5 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 7 \\ -1 \end{pmatrix} \\ &= \frac{1}{-17} \begin{pmatrix} -34 \\ -17 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{aligned}$$

$$\therefore H_p = \{(2, 1)\}$$

$$\begin{aligned} \text{d). } 2x - y &= 7 \\ 3x + 2y &= 14 \end{aligned} \Rightarrow \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 14 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 7 \\ 14 \end{pmatrix} \\ &= \frac{1}{4 - (-3)} \begin{pmatrix} 2 & 1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 14 \end{pmatrix} \\ &= \frac{1}{7} \begin{pmatrix} 28 \\ 7 \end{pmatrix} \end{aligned}$$

$$\Rightarrow H_p = \{(4, 1)\}$$

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$$7. e). \begin{cases} 5x - y = -29 \\ 2x + 3y = 2 \end{cases} \Rightarrow \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -29 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} -29 \\ 2 \end{pmatrix}$$

$$= \frac{1}{(5)(3) - (-1)(2)} \begin{pmatrix} 3 & 1 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} -29 \\ 2 \end{pmatrix}$$

$$= \frac{1}{17} \begin{pmatrix} -85 \\ 68 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$

$$\therefore HP = \{(-5, 4)\}$$

$$7. a). \begin{cases} x + 4y = -14 \\ 3x + 2y = -2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -14 \\ -2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -14 \\ -2 \end{pmatrix}$$

$$= \frac{1}{2 - 12} \begin{pmatrix} 2 & -4 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -14 \\ -2 \end{pmatrix}$$

$$= \frac{1}{-10} \begin{pmatrix} -20 \\ 40 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$\therefore x + y = 2 + (-4) = -2$$

$$b). \begin{cases} 3x - 4y = 18 \\ 5x + y = 7 \end{cases} \Rightarrow \begin{pmatrix} 3 & -4 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18 \\ 7 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 5 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 18 \\ 7 \end{pmatrix}$$

$$= \frac{1}{3 - (-20)} \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 18 \\ 7 \end{pmatrix}$$

$$= \frac{1}{23} \begin{pmatrix} 46 \\ -69 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\therefore x + y = 2 + (-3) = -1$$

$$c). \begin{cases} 2x + y = 12 \\ 3x - 2y = 25 \end{cases} \Rightarrow \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 25 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 12 \\ 25 \end{pmatrix}$$

$$= \frac{1}{3 - (-4)} \begin{pmatrix} -2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ 25 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} -49 \\ 14 \end{pmatrix}$$

$$= \begin{pmatrix} -7 \\ 2 \end{pmatrix}$$

$$\therefore x + y = -7 + 2 = -5$$

$$d). \begin{cases} 6x + 2y = 1 \\ 12x + 4y = 2 \end{cases} \Rightarrow \begin{pmatrix} 6 & 2 \\ 12 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 12 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \frac{1}{24 - 24} \begin{pmatrix} 4 & -2 \\ -12 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

karena didapatkan $\frac{1}{0}$ atau

determinan = 0

$\therefore x + y$ tidak bisa ditemukan dengan

informasi seperti di atas saja.

$$e). \begin{cases} 3x - 2y = 6 \\ 6x - 4y = 8 \end{cases} \Rightarrow \begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\text{karena } \det(A) = 3(-4) - (-2)(6) = 0$$

berarti

$\therefore x + y$ tidak bisa ditemukan dengan

informasi seperti di atas saja.

$$8. a). \begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} m & n \\ 1 & 1 \end{pmatrix}$$

$$\Leftrightarrow \frac{1}{-3 - (-6)} \begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} m & n \\ 1 & 1 \end{pmatrix}$$

$$\Leftrightarrow \frac{1}{3} \begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} m & n \\ 1 & 1 \end{pmatrix}$$

$$(45) \Rightarrow m = -\frac{1}{3} \text{ dan } n = -\frac{2}{3}$$

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$$b). 3p + 2q = 2 \Leftrightarrow 3p + 2q = 2$$

$$-3p = q \Leftrightarrow -3p - q = 0$$

$$\Rightarrow \begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -3 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -1 & -2 \\ 3 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2}{3} \\ 2 \end{pmatrix}$$

$$\therefore p = -\frac{2}{3} \text{ ; dan } q = 2.$$

9.a). $|A| = \begin{vmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{vmatrix}$

$$= (4 - 8 - 9) - (-12 + 6 - 4)$$

$$= -13 - (-10)$$

$$= -3$$

$$\text{adj } A = \begin{pmatrix} \begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} -4 & 3 \\ -3 & 1 \end{vmatrix} & \begin{vmatrix} -4 & 3 \\ 2 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ -2 & -3 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ -2 & -3 \end{vmatrix} & \begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -5 & -2 \\ 1 & 8 & 5 \\ 1 & 14 & 8 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-3} \begin{pmatrix} -1 & -5 & -2 \\ 1 & 8 & 5 \\ 1 & 14 & 8 \end{pmatrix}$$

$$b). 2x - 4y + 3z = 8$$

$$x + 2y - z = 5$$

$$-2x - 3y + z = -9$$

$$\Rightarrow \begin{pmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$$

$$= \frac{1}{-3} \begin{pmatrix} -1 & -5 & -2 \\ 1 & 8 & 5 \\ 1 & 14 & 8 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \\ -9 \end{pmatrix}$$

$$= -\frac{1}{3} \begin{pmatrix} -8 - 25 + 18 \\ 8 + 40 - 45 \\ 8 + 70 - 72 \end{pmatrix}$$

$$= -\frac{1}{3} \begin{pmatrix} -15 \\ 3 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix}$$

\therefore Himpunan penyelesaian

$$(x, y, z) = \{(5, -1, -2)\}$$

10a). $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{vmatrix}$

$$= (10 + 8 + 30) - (9 + 20 + 24)$$

$$= 56 - 53$$

$$= 3$$

$$\text{adj } A = \begin{pmatrix} \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} \end{pmatrix}$$

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$$\Rightarrow \text{adj } A = \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{3} \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix}$$

$$b). \begin{cases} x + 2y + 3z = 9 \\ 2x + 3y + 4z = 11 \\ x + 5y + 6z = 21 \end{cases}$$

$$2x + 3y + 4z = 11$$

$$x + 5y + 6z = 21$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 21 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 6 \end{pmatrix}^{-1} \begin{pmatrix} 9 \\ 11 \\ 21 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -2 & 3 & -1 \\ -8 & 3 & 2 \\ 7 & -3 & -1 \end{pmatrix} \begin{pmatrix} 9 \\ 11 \\ 21 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -18 + 33 - 21 \\ -72 + 33 + 42 \\ 63 - 33 - 21 \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} -6 \\ 3 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

\therefore Himpunan penyelesaiannya adalah

$$(x, y, z) = \{(-2, 1, 3)\}.$$

$$11. \begin{cases} x + y + z = 1 \\ 2x - y + 3z = 2 \\ 2x - y - z = 2 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 2 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\Rightarrow \circ D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 2 & -1 & -1 \end{vmatrix}$$

$$= (1 + 6 - 2) - (-2 - 3 - 2)$$

$$= 5 - (-7)$$

$$= 12$$

$$\circ D_x = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 2 & -1 & -1 \end{vmatrix} = D = 12.$$

$$\circ D_y = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 2 & 2 & -1 \end{vmatrix}$$

$$= (-2 + 6 + 4) - (4 + 6 - 2) = 0$$

$$\circ D_z = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= (-2 + 4 - 2) - (-2 - 2 + 4) = 0$$

$$\therefore x = \frac{12}{12} = 1$$

$$y = \frac{0}{12} = 0$$

$$z = \frac{0}{12} = 0$$

$$\Rightarrow H_p = \{(1, 0, 0)\}$$

$$12. \begin{cases} x + y + z = 2 \\ 2x - y + z = -1 \\ x - y - z = 0 \end{cases}$$

$$\begin{cases} 2x - y + z = -1 \\ x - y - z = 0 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= (1 + 1 - 2) - (-1 - 1 - 2) = 4$$

$$\circ D_x = \begin{vmatrix} 2 & 1 & 1 \\ -1 & -1 & 1 \\ 0 & -1 & -1 \end{vmatrix}$$

$$= (2 + 0 + 1) - (0 - 2 + 1) = 4$$

Ji Pemahaman Halaman 105

$$\circ D_y = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= (1+2+0) - (-1+0-4) = 8$$

$$\circ D_z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= (0-1-4) - (-2+1+0) = -4$$

$$\Rightarrow \begin{cases} x = \frac{4}{4} = 1 \\ y = \frac{8}{4} = 2 \\ z = \frac{-4}{4} = -1 \end{cases} \Rightarrow H_p = \{(1, 2, -1)\}$$

$$13. \begin{cases} x+2y-3z+4=0 \Leftrightarrow x+2y-3z=-4 \\ 2x-y+z-3=0 \Leftrightarrow 2x-y+z=3 \\ 3x+2y+z-10=0 \Leftrightarrow 3x+2y+z=10 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 10 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= (-1+6-12) - (9+2+4) = -22$$

$$\circ D_x = \begin{vmatrix} -4 & 2 & -3 \\ 3 & -1 & 1 \\ 10 & 2 & 1 \end{vmatrix}$$

$$= (4+20-18) - (30-0+6) = -22$$

$$\circ D_y = \begin{vmatrix} 1 & -4 & -3 \\ 2 & 3 & 1 \\ 3 & 10 & 1 \end{vmatrix}$$

$$= (3-12-60) - (-27+10-0) = -44$$

$$\circ D_z = \begin{vmatrix} 1 & 2 & -4 \\ 2 & -1 & 3 \\ 3 & 2 & 10 \end{vmatrix}$$

$$= (-10+18-16) - (12+6+40) = -66$$

$$\therefore H_p = \left\{ \left(\frac{-22}{-22}, \frac{-44}{-22}, \frac{-66}{-22} \right) \right\} = \{(1, 2, 3)\} \quad (48)$$

$$14. \begin{cases} x+y+2z=0 \\ 2x-2y+z=8 \\ 3x+2y+z=2 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 2 & -2 & 1 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ 2 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 1 & 1 & 2 \\ 2 & -2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= (-2+3+8) - (-12+2+2) = 17$$

$$\circ D_x = \begin{vmatrix} 0 & 1 & 2 \\ 8 & -2 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= (0+2+32) - (-8+0+0) = 34$$

$$\circ D_y = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 8 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= (8+0+0) - (48+2+0) = -34$$

$$\circ D_z = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -2 & 8 \\ 3 & 2 & 2 \end{vmatrix}$$

$$= (-4+24+0) - (0+16+4) = 0$$

$$\Rightarrow x = \frac{34}{17} = 2$$

$$y = \frac{-34}{17} = -2$$

$$z = \frac{0}{17} = 0$$

$$\Rightarrow H_p = \{(2, -2, 0)\}$$

$$15. \begin{cases} 4x-6y-z=19 \\ 2x+9y-6z=6 \\ -8x-3y+2z=13 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 4 & -6 & -1 \\ 2 & 9 & -6 \\ -8 & -3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 19 \\ 6 \\ -13 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 4 & -6 & -1 \\ 2 & 9 & -6 \\ -8 & -3 & 2 \end{vmatrix}$$

$$= (72-288+6) - (72+72-24) = -33$$

Uji Pemahaman Halaman 105

$$\bullet D_y = \begin{vmatrix} 4 & 19 & -1 \\ 2 & 6 & -6 \\ -8 & -13 & 2 \end{vmatrix}$$

$$= (48 + 9(2 + 26)) - (48 + 3(2 + 76))$$

$$= 550$$

$$\bullet D_x = \begin{vmatrix} 19 & -6 & -1 \\ 6 & 9 & -6 \\ -13 & -3 & 2 \end{vmatrix}$$

$$= (342 - 468 + 18) - (-117 + 342 - 72)$$

$$= -450 - 45$$

$$= -495$$

$$\bullet D_z = \begin{vmatrix} 4 & -6 & 19 \\ 2 & 9 & 6 \\ -8 & -3 & -13 \end{vmatrix}$$

$$= (-468 + 288 - 114) - (-1368 - 72 + 156)$$

$$= -294 - (-1284)$$

$$= 990$$

$$\therefore H_p = \left[\left(\frac{-495}{-330}, \frac{550}{-330}, \frac{990}{-330} \right) \right]$$

$$= \left\{ \left(\frac{3}{2}, -\frac{5}{3}, -3 \right) \right\}$$

$$16. \begin{cases} 2x - 6y + 3z = -6 \\ 3x + 2y - 2z = -14 \\ x + 4y + z = 12 \end{cases}$$

$$\begin{cases} 2x - 6y + 3z = -6 \\ 3x + 2y - 2z = -14 \\ x + 4y + z = 12 \end{cases}$$

$$\Rightarrow \begin{pmatrix} 2 & -6 & 3 \\ 3 & 2 & -2 \\ 1 & 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ -14 \\ 12 \end{pmatrix}$$

$$\bullet D = \begin{vmatrix} 2 & -6 & 3 \\ 3 & 2 & -2 \\ 1 & 4 & 1 \end{vmatrix}$$

$$= (4 + 12 + 36) - (6 - 16 - 18) = 80$$

$$\bullet D_x = \begin{vmatrix} -6 & -6 & 3 \\ -14 & 2 & -2 \\ 12 & 4 & 1 \end{vmatrix}$$

$$= (-12 + 144 - 168) - (72 + 48 + 84) \quad (49)$$

$$\Rightarrow D_x = -240$$

$$\bullet D_y = \begin{vmatrix} 2 & -6 & 3 \\ 3 & -14 & -2 \\ 1 & 12 & 1 \end{vmatrix}$$

$$= (-28 + 12 + 108) - (-42 - 48 - 18)$$

$$= 200$$

$$\bullet D_z = \begin{vmatrix} 2 & -6 & -6 \\ 3 & 2 & -14 \\ 1 & 4 & 12 \end{vmatrix}$$

$$= (48 + 84 - 72) - (-12 - 112 - 216)$$

$$= 400$$

$$H_p = \left[\left(\frac{-240}{80}, \frac{200}{80}, \frac{400}{80} \right) \right]$$

$$= \left\{ \left(-3, \frac{5}{2}, 5 \right) \right\}$$

$$17. \begin{cases} 2x - y + 3z = 11 \\ x - 3y + 2z = 12 \\ 3x + 2y - z = 17 \end{cases}$$

$$\begin{cases} 2x - y + 3z = 11 \\ x - 3y + 2z = 12 \\ 3x + 2y - z = 17 \end{cases}$$

$$3x + 2y - z = 17$$

$$\Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -3 & 2 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 12 \\ 17 \end{pmatrix}$$

$$\bullet D = \begin{vmatrix} 2 & -1 & 3 \\ 1 & -3 & 2 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= (6 - 6 + 6) - (-27 + 8 + 1) = 24$$

$$\bullet D_x = \begin{vmatrix} 11 & -1 & 3 \\ 12 & -3 & 2 \\ 17 & 2 & -1 \end{vmatrix}$$

$$= (33 - 34 + 72) - (-153 + 44 + 12)$$

$$= 168$$

$$\bullet D_y = \begin{vmatrix} 2 & 11 & 3 \\ 1 & 12 & 2 \\ 3 & 17 & -1 \end{vmatrix}$$

$$= (-24 + 66 + 51) - (108 + 68 - 11)$$

$$= 93 - 165$$

$$= -72$$

Jji Pemahaman Halaman 105

$$\circ D_2 = \begin{vmatrix} 2 & -1 & 11 \\ 1 & -3 & 12 \\ 3 & 2 & 17 \end{vmatrix}$$

$$= (-102 - 36 + 22) - (-99 + 48 - 17)$$

$$= -116 - (-68)$$

$$= -48$$

$$H_p = \left\{ \left(\frac{160}{24}, \frac{-72}{24}, \frac{-48}{24} \right) \right\} = \{(7, -3, -2)\}$$

$$18.a). \begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 6 \\ 1 & 2 & 1 \end{pmatrix} X = \begin{pmatrix} -2 \\ 6 \\ 1 \end{pmatrix}$$

$$\text{mis: } X = (a \ b \ c)^T$$

$$\Rightarrow \circ D = \begin{vmatrix} 2 & 2 & 3 \\ 2 & 1 & 6 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= (2 + 12 + 12) - (3 + 24 + 4) = -5$$

$$\circ D_x = \begin{vmatrix} -2 & 2 & 3 \\ 6 & 1 & 6 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= (-2 + 12 + 36) - (3 - 24 + 12) = 55$$

$$\circ D_y = \begin{vmatrix} 2 & -2 & 3 \\ 2 & 6 & 6 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (12 - 12 + 6) - (18 + 12 - 4) = -20$$

$$\circ D_z = \begin{vmatrix} 2 & 2 & -2 \\ 2 & 1 & 6 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= (2 + 12 - 8) - (-2 + 24 + 4) = -20$$

$$\Rightarrow a = \frac{55}{-5} = -11; b = \frac{-20}{-5} = 4; c = \frac{-20}{-5} = 4$$

$$\Rightarrow X = (a \ b \ c)^T$$

$$= \begin{pmatrix} -11 \\ 4 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -11 \\ 4 \\ 4 \end{pmatrix}$$

$$b). \begin{pmatrix} 6 & -4 \\ -3 & 3 \\ 5 & 2 \end{pmatrix} X = \begin{pmatrix} 2 \\ 6 \\ 39 \end{pmatrix}, \text{ misalkan } X = \begin{pmatrix} a \\ b \end{pmatrix}$$

\Rightarrow Untuk soal ini, maka akan menghilangkan 1 baris lalu hasil dari a, b akan dipastikan menggunakan baris yang dihilangkan tadi

$$\Rightarrow \begin{pmatrix} 6 & -4 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 6 & -4 \\ -3 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \frac{1}{18 - 12} \begin{pmatrix} 3 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 6 + 24 \\ 6 + 36 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 30 \\ 42 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

* Pemeriksaan

$$\begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} = (25 + 14) = (39)$$

karena $\begin{pmatrix} 5 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} = (39)$ dan sesuai dengan baris yang dihilangkan

$$\therefore X = \begin{pmatrix} 5 \\ 7 \end{pmatrix} //$$

$$19.a). x + 2y + 3z = 8$$

$$2x + 3y + 4z = 10$$

$$x + 5y + 7z = 20$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \\ 20 \end{pmatrix}$$

$$\circ D_A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{vmatrix}$$

$$= (21 + 8 + 30) - (9 + 20 + 28) = 2$$

Ji Pemahaman Halaman 65

$$\begin{aligned} \circ D_{Ax} &= \begin{vmatrix} 8 & 2 & 3 \\ 10 & 3 & 4 \\ 20 & 5 & 7 \end{vmatrix} \\ &= (160 + 160 + 150) - (180 + 160 + 140) \\ &= -2 \end{aligned}$$

$$\begin{aligned} \circ D_{Ay} &= \begin{vmatrix} 1 & 8 & 3 \\ 2 & 10 & 4 \\ 1 & 20 & 7 \end{vmatrix} \\ &= (70 + 32 + 120) - (30 + 80 + 112) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \circ D_{Az} &= \begin{vmatrix} 1 & 2 & 8 \\ 2 & 3 & 10 \\ 1 & 5 & 20 \end{vmatrix} \\ &= (60 + 20 + 80) - (24 + 50 + 80) \\ &= 6 \end{aligned}$$

$$\begin{aligned} \therefore x &= \frac{-2}{2} = -1 \\ y &= \frac{0}{2} = 0 \\ z &= \frac{6}{2} = 3 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow x = -1, y = 0, \text{ dan } z = 3$$

b). $a + b - c = 0$

$$-10a + 4b + 2c = 34$$

$$7a - 3b - c = -24$$

$$\Rightarrow \begin{pmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 34 \\ -24 \end{pmatrix}$$

$$\begin{aligned} \circ D_B &= \begin{vmatrix} 1 & 1 & -1 \\ -10 & 4 & 2 \\ 7 & -3 & -1 \end{vmatrix} \\ &= (-4 + 14 - 30) - (-28 - 6 + 10) = 4 \end{aligned}$$

$$\begin{aligned} \circ D_{Ba} &= \begin{vmatrix} 0 & 1 & -1 \\ 34 & 4 & 2 \\ -24 & -3 & -1 \end{vmatrix} \\ &= (0 - 48 + 102) - (96 + 0 - 34) \\ &= 54 - 62 \\ &= -8 \end{aligned}$$

$$\begin{aligned} \circ D_{Bb} &= \begin{vmatrix} 1 & 0 & -1 \\ -10 & 34 & 2 \\ 7 & -24 & -1 \end{vmatrix} \\ &= (-34 + 0 - 240) - (-238 - 48 + 0) \\ &= 12 \end{aligned}$$

$$\begin{aligned} \circ D_{Bc} &= \begin{vmatrix} 1 & 1 & 0 \\ -10 & 4 & 34 \\ 7 & -3 & -24 \end{vmatrix} \\ &= (-96 + 238 + 0) - (0 - 102 + 240) \\ &= 4 \end{aligned}$$

$$\begin{aligned} \Rightarrow a &= \frac{-8}{4} = -2 \\ b &= \frac{12}{4} = 3 \\ c &= \frac{4}{4} = 1 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow a = -2; b = 3; \text{ dan } c = 1 //$$

$$20. \begin{pmatrix} 3-a & 3 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} -3 \\ x \end{pmatrix} = \begin{pmatrix} -3 \\ x \end{pmatrix}$$

o Baris ke-2.

$$\Rightarrow 3 - 2x = x \Rightarrow 3x = 3 \Leftrightarrow x = 1$$

o Baris ke-1

$$\Rightarrow (3-a)(-3) + 3x = -3$$

$$\Rightarrow -9 + 3a + 3(1) = -3$$

$$\Leftrightarrow 3a = (9-3) - 3$$

$$\Leftrightarrow 3a = 3$$

$$\Leftrightarrow a = 1 //$$

$$\therefore a = 1 \text{ dan } x = 1$$

21. $f(A, B) = BA - B$

$$\begin{aligned} &= \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 11 & 7 \\ 7 & 4 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 4 \\ 5 & 3 \end{pmatrix} \end{aligned}$$

1. Pemahaman Halaman 105

$$2. \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} + x \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} + x \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ 2 & 12 \end{pmatrix}$$

$$x \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ 2 & 12 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$$

$$x \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 7 \\ 3 & 8 \end{pmatrix}$$

$$x = \begin{pmatrix} 2 & 7 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & 7 \\ 3 & 8 \end{pmatrix} \cdot \frac{1}{6-5} \cdot \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 7 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$$

$$\Rightarrow x = \begin{pmatrix} -31 & 19 \\ -34 & 21 \end{pmatrix}$$

23. Pembayaran $\Rightarrow 4x + 2y = 510.000$
Chalid

• Pembayaran $\Rightarrow 3x + 5y = 715.000$
Andika

• Model matriks

$$\begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 510.000 \\ 715.000 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}^{-1} \begin{pmatrix} 510.000 \\ 715.000 \end{pmatrix}$$

$$= \frac{1}{20-6} \begin{pmatrix} 5 & -2 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 510.000 \\ 715.000 \end{pmatrix}$$

$$= \frac{1}{14} \begin{pmatrix} 2.550.000 - 1.430.000 \\ -1.530.000 + 2.860.000 \end{pmatrix}$$

$$= \frac{1}{14} \begin{pmatrix} 1.120.000 \\ 1.330.000 \end{pmatrix}$$

$$= \begin{pmatrix} 80.000 \\ 95.000 \end{pmatrix}$$

\therefore Harga 1 kemeja = Rp 80.000,00

Harga 1 Celana = Rp 95.000,00

24. Misalkan

• Harga 1 kg anggur = x

• Harga 1 kg mangga = y

• Harga 1 kg jeruk = z.

\Rightarrow Pembelian Arman $\Rightarrow 2x + 5y + 3z = 265.000$

• Pembelian Dewa $\Rightarrow 3x + 2y + 2z = 225.000$

• Pembelian Thomas $\Rightarrow 4x + 3y + 4z = 353.000$

• Model matriks.

$$\begin{pmatrix} 2 & 5 & 3 \\ 3 & 2 & 2 \\ 4 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 265.000 \\ 225.000 \\ 353.000 \end{pmatrix}$$

$$\circ D = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 2 & 2 \\ 4 & 3 & 4 \end{vmatrix}$$

$$= (16 + 40 + 27) - (24 + 12 + 60)$$

$$= 83 - 96$$

$$= -13$$

$$\circ D_x = \begin{vmatrix} 265.000 & 5 & 3 \\ 225.000 & 2 & 2 \\ 353.000 & 3 & 4 \end{vmatrix}$$

$$= (2.120.000 + 3.530.000 + 2.025.000)$$

$$- (2.118.000 + 1.590.000 + 4.500.000)$$

$$= -533.000$$

$$\circ D_y = \begin{vmatrix} 2 & 265.000 & 3 \\ 3 & 225.000 & 2 \\ 4 & 353.000 & 4 \end{vmatrix}$$

$$= (1.800.000 + 2.120.000 + 3.177.000)$$

$$- (2.700.000 + 1.412.000 + 3.180.000)$$

$$= -195.000$$

Jji Pemahaman Halaman 105

$$\bullet D_2 = \begin{vmatrix} 2 & 5 & 265.000 \\ 3 & 2 & 225.000 \\ 4 & 3 & 353.000 \end{vmatrix}$$

$$\begin{aligned} &= (1 \cdot 4 (2.000 + 4.500.000 + 2.385.000) \\ &\quad - (2.120.000 + 1.350.000 + 5.295.000)) \\ &= -468.000 \end{aligned}$$

∴ Harga 1 kg anggur

$$= x = \frac{D_x}{D} = \frac{-533.000}{-13} = \text{Rp } 41.000,00$$

• Harga 1 kg mangga

$$= y = \frac{D_y}{D} = \frac{-195.000}{-13} = \text{Rp } 15.000,00$$

• Harga 1 kg jeruk

$$= z = \frac{D_z}{D} = \frac{-468.000}{-13} = \text{Rp } 36.000,00$$

25. Misalkan

□ Harga 1 liter beras = x

□ Harga 1 liter minyak goreng = y .

$$\Rightarrow 3x + 4y = 86.000$$

$$4x + 2y = 68.000$$

$$\Rightarrow \begin{pmatrix} 3 & 4 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 86.000 \\ 68.000 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 4 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 86.000 \\ 68.000 \end{pmatrix}$$

$$= \frac{1}{6-16} \begin{pmatrix} 2 & -4 \\ -4 & 3 \end{pmatrix} \begin{pmatrix} 86.000 \\ 68.000 \end{pmatrix}$$

$$= \frac{1}{-10} \begin{pmatrix} 2 \cdot 86.000 - 4 \cdot 68.000 \\ -4 \cdot 86.000 + 3 \cdot 68.000 \end{pmatrix}$$

$$= -\frac{1}{10} \begin{pmatrix} -100.000 \\ -140.000 \end{pmatrix}$$

$$= \begin{pmatrix} 10.000 \\ 14.000 \end{pmatrix}$$

∴ Karena Rosa membeli 2 liter beras dan 1 liter minyak goreng.

$$\begin{aligned} \Rightarrow \text{Total pembayaran Rosa} \\ &= 2 \cdot 10.000 + 1 \cdot 14.000 \\ &= \text{Rp } 34.000,00 \end{aligned}$$

Soal Pendalaman Halaman 54

$$\begin{pmatrix} a+3 & 2b-6 \\ b-a & -3d+4 \end{pmatrix} + \begin{pmatrix} b+d & 2c+d \\ 4 & 3b+1 \end{pmatrix}^T = \begin{pmatrix} 2 & 8 \\ 9 & 32 \end{pmatrix}$$

$$\begin{pmatrix} a+3 & 2b-6 \\ b-a & -3d+4 \end{pmatrix} + \begin{pmatrix} b+d & 4 \\ 2c+d & 3b+1 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 9 & 32 \end{pmatrix}$$

$$\bullet (a+3) + (b+d) = 2$$

$$\Leftrightarrow a + b + d = -1 \quad \dots \dots \dots (1)$$

$$\bullet (2b-6) + 4 = 8$$

$$\Leftrightarrow 2b = 10 \Leftrightarrow b = 5 \quad \checkmark$$

$$\bullet (-3d+4) + (3b+1) = 32$$

$$\Leftrightarrow 3d = 3b - 27 = 3(5) - 27 = -12$$

$$\Leftrightarrow d = -4$$

$$\bullet (1) \Rightarrow a + b + d = -1$$

$$\Leftrightarrow a = -1 - (b+d) = -1 - (5 + (-4))$$

$$\Leftrightarrow a = -2$$

$$\bullet (b-a) + (2c+d) = 9$$

$$2c = 9 + a - b - d = 9 + (-2) - 5 - (-4) = 6$$

$$\Leftrightarrow c = 3$$

$$\therefore a + b + c + d$$

$$= -2 + 5 + 3 + (-4)$$

$$= 2.$$

Jawaban: D.

Soal Pendalaman Halaman 77.

$$\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} X = \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix} \right] X = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix}$$

$$X = \frac{1}{2-3} \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ 3 & 1 \end{pmatrix}$$

$$= (-1) \cdot \begin{pmatrix} 13 & -3 \\ 18 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} -13 & 3 \\ -18 & 5 \end{pmatrix}.$$

$$\Rightarrow \det(X) = (-13) \cdot 5 - (3) \cdot (-18)$$

$$= -65 + 54$$

$$= -11$$

Jawaban: B.

Latihan Soal Akhir Bab 2

$$\boxed{1} \quad \begin{pmatrix} 4 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 2 & 8 \\ 9 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 3a-6 \\ 2a & 5 \end{pmatrix}$$

$$\Rightarrow a+8 = 3a-6$$

$$14 = 2a$$

$$7 = a$$

Jawaban : E

$$\boxed{2} \quad A_{2 \times 2} \times B_{2 \times 3} = AB_{2 \times 3}$$

Jawaban : C

$$\boxed{3} \quad \det A = 0$$

$$12 - (-b)x = 0$$

$$6x = -12$$

$$x = -2$$

$$\Rightarrow A = \begin{pmatrix} 4 & -6 \\ x & 3 \end{pmatrix}$$

Jawaban : A

$$\boxed{4} \quad \begin{pmatrix} 3 & 6 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} 1 & 3x \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 33 & -12 \\ -20 & -16 \end{pmatrix}$$

$$\Rightarrow 3(3x) + 6(4) = -12$$

$$9x + 24 = -12$$

$$9x = -36$$

$$x = -4$$

Jawaban : E

$$\boxed{5} \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$z = x - y = 12 - 5 = 7$$

Jawaban : A

$$\boxed{6} \quad \Rightarrow 1+b = -3$$

$$b = -4$$

$$\Rightarrow a+b+10 = 9$$

$$a+(-4)+10 = 9$$

$$a+b = 9$$

$$a = 3$$

Jawaban : C

$$\boxed{7} \quad 2A = 2 \begin{pmatrix} 2 & x \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 2x \\ 8 & 10 \end{pmatrix}$$

$$\Rightarrow 2A - B = C$$

$$\begin{pmatrix} 4 & 2x \\ 8 & 10 \end{pmatrix} - \begin{pmatrix} -3 & 6 \\ y & 5 \end{pmatrix} = \begin{pmatrix} z & -2 \\ 3 & 5 \end{pmatrix}$$

$$\Rightarrow 4 - (-3) = z$$

$$7 = z$$

$$\Rightarrow 8 - y = 3$$

$$-y = -5$$

$$y = 5$$

$$\Rightarrow 2x - 6 = -2$$

$$2x = 4$$

$$x = 2$$

$$\Rightarrow x + y + z = 2 + 5 + 7$$

$$= 14$$

Jawaban : D

$$\boxed{8} \quad A + B = C^T$$

$$\begin{pmatrix} 2 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 \\ b+1 & c \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 5 & 4 \end{pmatrix}$$

$$\Rightarrow a+4 = 0$$

$$a = -4$$

$$\Rightarrow 1+c = 4$$

$$c = 3$$

$$\Rightarrow b+b+1 = 5$$

$$2b = 4$$

$$b = 2$$

$$\Rightarrow a - 2b + c = -4 - 2(2) + 3$$

$$= -4 - 4 + 3$$

$$= -5$$

Jawaban : B

$$\boxed{9.} \begin{pmatrix} 4 & 3 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 2 & -5 \end{pmatrix}^{-1} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$= \frac{1}{-26} \begin{pmatrix} -5 & -3 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{k} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{array}{l|l} k = -26 & ab - cd - k = (-5)(-3) - (-2)(4) - (-26) \\ a = -5 & = 15 + 8 + 26 \\ b = -3 & = 49 \\ c = -2 & \\ d = 4 & \end{array}$$

Jawaban : E

$$\boxed{10.} \begin{pmatrix} 2k & 3k \\ -k & 4k \end{pmatrix} \begin{pmatrix} 4 & h \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow 2k(4) + 3k(1) = 1$$

$$8k + 3k = 1$$

$$11k = 1$$

$$k = \frac{1}{11}$$

$$\Rightarrow 2k(h) + 3k(2) = 0$$

$$\frac{2h}{11} + \frac{6}{11} = 0$$

$$\frac{2h}{11} = -\frac{6}{11}$$

$$h = -3$$

$$\Rightarrow \frac{h}{k} = \frac{-3}{\frac{1}{11}} = -33$$

Jawaban : A

$$\boxed{11.} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -12 & 6 \\ p & -3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\Rightarrow 3(-12) + 4(p) = 0$$

$$4p = 36$$

$$p = 9$$

Jawaban : E

$$\boxed{12.} \begin{vmatrix} 2 & -2 \\ k^3 & 8 \end{vmatrix} = 0$$

\Rightarrow tidak punya invers
maka $\det = 0$

$$16 + 2k^3 = 0$$

$$2k^3 = -16$$

$$k^3 = -8$$

$$k = -2$$

Jawaban : C

$$\boxed{13.} \begin{vmatrix} x & x \\ -6 & -3x \end{vmatrix} = 0$$

$$-3x^2 - (-6x) = 0$$

$$-3x^2 + 6x = 0$$

$$-3x(x-2) = 0$$

$$\Rightarrow x = 0 \text{ atau } x = 2$$

Jawaban : B

$$\boxed{14.} A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}, A^2 = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 8 & 0 \\ 0 & 27 \end{pmatrix}$$

$$\begin{aligned} \Rightarrow A^3 + A^2 &= \begin{pmatrix} 8 & 0 \\ 0 & 27 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix} \\ &= \begin{pmatrix} 12 & 0 \\ 0 & 36 \end{pmatrix} \end{aligned}$$

Jawaban : D

$$15) A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}, A^T = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{6-4} \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$$

$$\Rightarrow k \cdot |A^T| = |A^{-1}|$$

$$k(6-4) = \left(\frac{3}{2} \cdot \frac{2}{2} - \left(-\frac{1}{2} \right) \left(-\frac{4}{2} \right) \right)$$

$$k \cdot 2 = \frac{3}{2} - 1$$

$$2k = \frac{1}{2}$$

$$k = \frac{1}{4}$$

Jawaban: A

$$16) \begin{vmatrix} x & 1 \\ 5 & -2x \end{vmatrix} = \begin{vmatrix} x & x \\ 8 & x \end{vmatrix}$$

$$(-2x^2 - 5) = (x^2 - 8x)$$

$$3x^2 - 8x + 5 = 0$$

$$(3x-5)(x-1) = 0$$

$$x = \frac{5}{3} \text{ atau } x = 1$$

Jawaban: E

$$17) \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{16} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 15 \\ 1 \end{pmatrix}$$

$$\therefore a = 4, b = -2, c = -5, d = 3, k = 2$$

$$\Rightarrow ab - cd - k = -8 + 15 - 2 = 5$$

Jawaban: C

$$18) k \begin{pmatrix} 3 & 9 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} -2 & h \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3k & 9k \\ 3k & 6k \end{pmatrix} \begin{pmatrix} -2 & h \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -6k+9k & 3kh-9k \\ -6k+6k & 3kh-6k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow 3k = 1$$

$$k = \frac{1}{3}$$

$$\Rightarrow 3kh - 9k = 0$$

$$h - 3 = 0$$

$$h = 3$$

$$\Rightarrow \frac{h}{k} = \frac{3}{\frac{1}{3}} = 9$$

Jawaban: E

$$19) M = \begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 & 9 \\ 1 & 8 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 8 & -9 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

$$\Rightarrow M \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}$$

Jawaban: E

$$20) \begin{vmatrix} 2x & 6 \\ -1 & x \end{vmatrix} = \begin{vmatrix} x & 5 \\ x & 3x \end{vmatrix}$$

$$2x^2 + 6 = 3x^2 - 5x$$

$$0 = x^2 - 5x - 6$$

$$0 = (x-6)(x+1)$$

$$x = 6 \text{ atau } x = -1$$

Jawaban: B

$$\boxed{21} \begin{vmatrix} p-1 & p \\ 1-q & q \end{vmatrix} = 0$$

$$\Rightarrow (p-1)q - (1-q)p = 0$$

$$pq - q - (p - pq) = 0$$

$$2pq - p - q = 0$$

$$2pq = p + q$$

$$\Rightarrow 2x^2 + x - a = 0 \quad \Rightarrow x_1 = p, x_2 = q$$

$$p + q = -\frac{b}{a} = -\frac{1}{2}$$

$$pq = \frac{c}{a} = -\frac{a}{2}$$

$$\Rightarrow 2pq = p + q$$

$$2\left(-\frac{a}{2}\right) = -\frac{1}{2}$$

$$-a = -\frac{1}{2}$$

$$a = \frac{1}{2}$$

Jawaban : C

$$\boxed{22} \quad A X = B$$

$$X = A^{-1} \cdot B$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \frac{1}{4-6} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -8+2 & -6+1 \\ 6-1 & \frac{9}{2}-\frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} -6 & -5 \\ 5 & 4 \end{pmatrix}$$

Jawaban : E

$$\boxed{23} \quad A X = B$$

$$X = A^{-1} B$$

$$X = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 3 & 18 \\ 1 & 11 \end{pmatrix}$$

$$= \frac{1}{-4-6} \begin{pmatrix} -1 & -2 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 18 \\ 1 & 11 \end{pmatrix}$$

$$= -\frac{1}{10} \begin{pmatrix} -3-2 & -18-22 \\ -9+4 & -54+44 \end{pmatrix}$$

$$= -\frac{1}{10} \begin{pmatrix} -5 & -40 \\ -5 & -10 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 4 \\ \frac{1}{2} & 1 \end{pmatrix}$$

Jawaban : C

$$\boxed{24} \quad P = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 5 & 13 \\ 4 & 10 \end{pmatrix}$$

$$= \frac{1}{2-3} \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 13 \\ 4 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5 & 13 \\ 4 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -10+12 & -26+30 \\ 5-4 & 13-10 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$\Rightarrow P \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2+4 \\ 1+3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

Jawaban : D

$$\begin{aligned}
 \boxed{25} \quad X &= \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \\
 &= \frac{1}{2-1} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} 1-1 & 3-4 \\ -1+2 & -3+8 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & -1 \\ 1 & 5 \end{pmatrix}
 \end{aligned}$$

Jawaban : B

$$\begin{aligned}
 \boxed{26} \quad p &= \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \\
 p^T &= \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix} \\
 \Rightarrow (p^T)^{-1} &= \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}^{-1} \\
 &= \frac{1}{12-10} \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix} \\
 &= \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix}
 \end{aligned}$$

Jawaban : B

$$\begin{aligned}
 \boxed{27} \quad A &= \begin{pmatrix} -5 & 6 \\ 11 & 5 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 4 \end{pmatrix}^{-1} \\
 &= \begin{pmatrix} -5 & 6 \\ 11 & 5 \end{pmatrix} \frac{1}{12+1} \begin{pmatrix} 4 & 1 \\ -1 & 3 \end{pmatrix} \\
 &= \frac{1}{13} \begin{pmatrix} -5 & 6 \\ 11 & 5 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ -1 & 3 \end{pmatrix} \\
 &= \frac{1}{13} \begin{pmatrix} -20-6 & -5+18 \\ 44-5 & 11+15 \end{pmatrix} \\
 &= \frac{1}{13} \begin{pmatrix} -26 & 13 \\ 39 & 26 \end{pmatrix} \\
 &= \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}
 \end{aligned}$$

Jawaban : A

$$\begin{aligned}
 \boxed{28} \quad AX &= B \\
 X &= A^{-1} B \\
 X &= \begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix}^{-1} \begin{pmatrix} 17 & 21 \\ -2 & -18 \end{pmatrix} \\
 &= \frac{1}{-6-2} \begin{pmatrix} -2 & -1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 17 & 21 \\ -2 & -18 \end{pmatrix} \\
 &= -\frac{1}{8} \begin{pmatrix} -34+2 & -42+18 \\ -34-6 & -42-54 \end{pmatrix} \\
 &= -\frac{1}{8} \begin{pmatrix} -32 & -24 \\ -40 & -96 \end{pmatrix} \\
 &= \begin{pmatrix} 4 & 3 \\ 5 & 12 \end{pmatrix}
 \end{aligned}$$

Jawaban : A

$$\begin{aligned}
 \boxed{29} \quad A \times A^{-1} &= B \\
 X &= A^{-1} B A \\
 &= \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 3 & 2 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix} \\
 &= \frac{1}{6+2} \begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 6+2 & -6+6 \\ -2-2 & 2-6 \end{pmatrix} \\
 &= \frac{1}{8} \begin{pmatrix} 3 & 2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 8 & 0 \\ -4 & -4 \end{pmatrix} \\
 &= \frac{1}{8} \begin{pmatrix} 24-8 & -8 \\ -8-8 & -8 \end{pmatrix} \\
 &= \frac{1}{8} \begin{pmatrix} 16 & -8 \\ -16 & -8 \end{pmatrix} \\
 &= \begin{pmatrix} 2 & -1 \\ -2 & -1 \end{pmatrix}
 \end{aligned}$$

Jawaban : D

$$\boxed{30} \quad \begin{vmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{vmatrix} = 0$$

$$\Rightarrow t-2 \begin{vmatrix} t+1 & -2 \\ 0 & t-4 \end{vmatrix} - 4 \begin{vmatrix} 1 & -2 \\ 0 & t-4 \end{vmatrix} + 3 \begin{vmatrix} 1 & t+1 \\ 0 & 0 \end{vmatrix} = 0$$

$$t-2((t+1)(t-4)-0) - 4((t-4)-0) + 0 = 0$$

$$(t-2)(t^2-3t-4) - 4t + 16 = 0$$

$$t^3 - 3t^2 - 4t - 2t^2 + 6t + 8 - 4t + 16 = 0$$

$$t^3 - 5t^2 - 2t + 24 = 0$$

$$(t+2)(t-3)(t-4) = 0$$

$$t_1 = -2 \quad t_2 = 3 \quad t_3 = 4$$

$$\Rightarrow t_1 \cdot t_2 \cdot t_3 = -2 \cdot 3 \cdot 4$$

$$= -24$$

Jawaban : A

$$\boxed{31} \quad \begin{vmatrix} 3 & 2 & 4 \\ 1 & 5 & 3 \\ -1 & 6 & 2 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 5 & 3 \\ 6 & 2 \end{vmatrix} - 2 \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 5 \\ -1 & 6 \end{vmatrix}$$

$$= 3(10-18) - 2(2+3) + 4(6+5)$$

$$= 3(-8) - 2(5) + 4(11)$$

$$= -24 - 10 + 44$$

$$= 10$$

Jawaban : B

$$\boxed{32} \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 1 \end{pmatrix}$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 8 & 9 \end{vmatrix} = 9 - 16 = -7$$

Jawaban : A

$$\boxed{33} \quad A = \begin{pmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 6 \end{pmatrix}$$

$$\Rightarrow \alpha_{11} = (-1)^{1+1} M_{11} = 1(-22) = -22$$

$$\alpha_{12} = (-1)^{1+2} M_{12} = -1(-2) = 2$$

$$\alpha_{13} = (-1)^{1+3} M_{13} = 1(4) = 4$$

$$\alpha_{21} = (-1)^{2+1} M_{21} = -1(14) = -14$$

$$\alpha_{22} = (-1)^{2+2} M_{22} = 1(16) = 16$$

$$\alpha_{23} = (-1)^{2+3} M_{23} = -1(-5) = 5$$

$$\alpha_{31} = (-1)^{3+1} M_{31} = 1(-10) = -10$$

$$\alpha_{32} = (-1)^{3+2} M_{32} = -1(4) = -4$$

$$\alpha_{33} = (-1)^{3+3} M_{33} = 1(-8) = -8$$

$$\text{Adj } A = \begin{pmatrix} -22 & -14 & -10 \\ 2 & 16 & -4 \\ 4 & 5 & -8 \end{pmatrix}$$

Jawaban : E

$$\boxed{34} \quad AB = \begin{pmatrix} 2 & -4 & 3 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & 2 \\ -1 & -8 & -5 \\ -1 & -14 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 2+4-3 & 10+32-42 & 4+20-24 \\ 1-2+1 & 5-16+14 & 2-10+8 \\ -2+3-1 & -10+24-14 & -4+15-8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$= 3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Jawaban : D

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$$\begin{vmatrix} 2 & k+3 & 1 \\ 2k & 4 & -3 \\ 2 & 5 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2 \begin{vmatrix} 4 & -3 \\ 5 & 1 \end{vmatrix} - (k+3) \begin{vmatrix} 2k & -3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 2k & 4 \\ 2 & 5 \end{vmatrix} = 0$$

$$2(4+15) - (k+3)(2k+6) + 10k - 8 = 0$$

$$8 + 30 - (2k^2 + 12k + 18) + 10k - 8 = 0$$

$$-2k^2 - 2k + 12 = 0$$

$$-2(k^2 + k - 12) = 0$$

$$-2(k+3)(k-2) = 0$$

\therefore nilai k yang memenuhi adalah $k = -3$

atau $k = 2$

Jawaban : D

$$36. \text{ garis I : } -5x + \frac{5}{3}y = -\frac{25}{3}$$

$$-15x + 5y = -25$$

$$-3x + y = -5$$

$$\text{garis II : } -\frac{12}{5}x + 6y = -\frac{72}{5}$$

$$-12x + 30y = -72$$

$$-2x + 5y = -12$$

$$\text{Pers I : } 3x - y = 5$$

$$\text{II : } 2x - 5y = 12$$

$$D = \begin{vmatrix} 3 & -1 \\ 2 & -5 \end{vmatrix} = -15 + 2 = -13$$

$$D_x = \begin{vmatrix} 5 & -1 \\ 12 & -5 \end{vmatrix}$$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 5 & -1 \\ 12 & -5 \end{vmatrix}}{-13}$$

Jawaban : D

$$37. x + 2y = 6$$

$$2x + 3y = 11$$

$$\Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

$$= \frac{1}{3-4} \begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 6 \\ 11 \end{pmatrix}$$

Jawaban : B

$$38. x = \text{pegawai pria}$$

$$y = \text{pegawai wanita}$$

$$\Rightarrow x = \frac{2}{5}y \quad \left| \begin{array}{l} \Rightarrow y - 12 = x \\ x - y = -12 \end{array} \right.$$

$$5x = 2y$$

$$5x - 2y = 0$$

$$\Rightarrow \begin{pmatrix} 5 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -12 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -12 \end{pmatrix}$$

$$= \frac{1}{-5+2} \begin{pmatrix} -1 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 0 \\ -12 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

Jawaban : E

39] misal $x = \text{kecap manis}$

$y = \text{kecap asin}$

$$\Rightarrow x + y = 75.000$$

$$5x + 2y = 300$$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 75.000 \\ 300 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 5 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 75.000 \\ 300 \end{pmatrix}$$

$$= \frac{1}{2-5} \begin{pmatrix} 2 & -1 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 75.000 \\ 300 \end{pmatrix}$$

$$= -\frac{1}{3} \begin{pmatrix} 2 & -1 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 75.000 \\ 300 \end{pmatrix}$$

Jawaban: A

40.] Pondasi $\rightarrow 0,1x + 0,1y + 0,3z = 68$

$$x + y + 3z = 680$$

Pemasangan $\rightarrow 0,3x + 0,2y + 0,4z = 116$

$$3x + 2y + 4z = 1160$$

Finishing $\rightarrow 0,1x + 0,2y + 0,1z = 51$

$$x + 2y + z = 510$$

$$\Rightarrow x + y + 3z = 680$$

$$3x + 2y + 4z = 1160$$

$$x + 2y + z = 510$$

$$\begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 4 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 680 \\ 1160 \\ 510 \end{pmatrix}$$

Jawaban: D

B. Uraian

$$1] 2A = \begin{pmatrix} 4 & 6 & -2 \\ 6 & 0 & -8 \end{pmatrix}$$

$$3B = 3 \begin{pmatrix} 0 & 1 & -2 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 3 & -6 \\ 3 & -3 & 6 \end{pmatrix}$$

$$4C = 4 \begin{pmatrix} 3 & 0 & 1 \\ 0 & -1 & 5 \end{pmatrix} = \begin{pmatrix} 12 & 0 & 4 \\ 0 & -4 & 20 \end{pmatrix}$$

$$\Rightarrow 2A + 3B - 4C$$

$$= \begin{pmatrix} 4 & 6 & -2 \\ 6 & 0 & -8 \end{pmatrix} + \begin{pmatrix} 0 & 3 & -6 \\ 3 & -3 & 6 \end{pmatrix} - \begin{pmatrix} 12 & 0 & 4 \\ 0 & -4 & 20 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & 9 & -12 \\ 9 & 1 & -22 \end{pmatrix}$$

$$2] A = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}, B = \begin{pmatrix} 6 & 1 \\ 4 & -3 \end{pmatrix}, C = \begin{pmatrix} 0 & -3 \\ -1 & 2 \end{pmatrix}$$

$$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$a) A+B = \begin{pmatrix} 9 & 1 \\ 3 & 1 \end{pmatrix}$$

$$b) A+C = \begin{pmatrix} 3 & -3 \\ -2 & 6 \end{pmatrix}$$

$$c) B+C = \begin{pmatrix} 6 & -2 \\ 3 & -1 \end{pmatrix}$$

$$d) A+(B+C) = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix} + \begin{pmatrix} 6 & -2 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 9 & -2 \\ 2 & 3 \end{pmatrix}$$

$$e) (A+B)+C = \begin{pmatrix} 9 & 1 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 9 & -2 \\ 2 & 3 \end{pmatrix}$$

$$f) A+0 = A = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}$$

$$g) B+0 = B = \begin{pmatrix} 6 & 1 \\ 4 & -3 \end{pmatrix}$$

$$h) C+0 = C = \begin{pmatrix} 0 & -3 \\ -1 & 2 \end{pmatrix}$$

$$\boxed{3} \text{ a.) } \begin{pmatrix} -2 & 5 & 0 \\ 1 & 0 & -1 \end{pmatrix} - \begin{pmatrix} -7 & 1 & 4 \\ 6 & -2 & 0 \end{pmatrix} \\ = \begin{pmatrix} 5 & 4 & -4 \\ -5 & 2 & -1 \end{pmatrix}$$

$$\text{b.) } \begin{pmatrix} 3 & 2 \\ -1 & 0 \\ 6 & -4 \end{pmatrix} - \begin{pmatrix} -2 & 0 \\ 1 & 3 \\ 5 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ -2 & -3 \\ 1 & -3 \end{pmatrix}$$

$$\text{c.) } \begin{pmatrix} 8 \\ -2 \\ -2 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

$$\text{d.) } \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 4 \\ -5 & 7 \end{pmatrix} = \begin{pmatrix} 1 & -7 \\ 6 & -5 \end{pmatrix}$$

$$\text{e.) } \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

$$\text{f.) } \begin{pmatrix} 5 & 3 & 4 & 6 \\ 4 & 2 & 5 & 3 \\ 6 & 4 & 3 & 7 \end{pmatrix} - \begin{pmatrix} 1,7 & 0,9 & 1,0 & 4,8 \\ 0,5 & 0,1 & 0,8 & 2,6 \\ 1,2 & 0,8 & 0,6 & 1,3 \end{pmatrix} \\ = \begin{pmatrix} 3,3 & 2,1 & 3,0 & 1,2 \\ 3,5 & 1,9 & 4,2 & 0,4 \\ 4,8 & 3,2 & 2,4 & 5,7 \end{pmatrix}$$

$$\boxed{4} \text{ } A = \begin{pmatrix} 5 & -1 \\ 0 & 3 \\ -4 & 2 \end{pmatrix}, B = \begin{pmatrix} -2 & 8 \\ 5 & 10 \\ 0 & -7 \end{pmatrix}, C = \begin{pmatrix} 6 & -3 \\ -8 & 11 \\ -7 & 0 \end{pmatrix}$$

$$\text{a.) (i) } A - B - C = \begin{pmatrix} 1 & -6 \\ 3 & -18 \\ 3 & 9 \end{pmatrix}$$

$$\text{(ii) } A - (B + C) = \begin{pmatrix} 5 & -1 \\ 0 & 3 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 5 \\ -3 & 21 \\ -7 & -7 \end{pmatrix} \\ = \begin{pmatrix} 1 & -6 \\ 3 & -18 \\ 3 & 9 \end{pmatrix}$$

$$\text{iii) } (A+B) - C = \begin{pmatrix} 3 & 7 \\ 5 & 13 \\ -4 & -5 \end{pmatrix} - \begin{pmatrix} 6 & -3 \\ -8 & 11 \\ -7 & 0 \end{pmatrix} \\ = \begin{pmatrix} -3 & 10 \\ 13 & -2 \\ 3 & -5 \end{pmatrix}$$

$$\text{iv) } A - C = \begin{pmatrix} -1 & 2 \\ 8 & -8 \\ 3 & 2 \end{pmatrix}$$

$$\text{v) } B - C = \begin{pmatrix} -8 & 11 \\ 13 & -1 \\ 7 & -7 \end{pmatrix}$$

$$\text{vi) } A - (B - C) = \begin{pmatrix} 5 & -1 \\ 0 & 3 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} -8 & 11 \\ 13 & -1 \\ 7 & -7 \end{pmatrix} \\ = \begin{pmatrix} 13 & -12 \\ -13 & 4 \\ -11 & 9 \end{pmatrix}$$

$$\text{b.) (i) } A - B = \begin{pmatrix} 7 & -9 \\ -5 & -7 \\ -4 & 9 \end{pmatrix}$$

$\therefore A - B \neq B - C$
Pernyataan (i) salah

(ii) $B - C = C - B$ } Pengurangan matriks
(iii) $A - C = C - A$ } tidak bersifat komutatif
jadi pernyataan salah

$$\text{(iv) } (A - B) - C = \begin{pmatrix} 7 & -9 \\ -5 & -7 \\ -4 & 9 \end{pmatrix} - \begin{pmatrix} 6 & -3 \\ -8 & 11 \\ -7 & 0 \end{pmatrix} \\ = \begin{pmatrix} 1 & -6 \\ 3 & -18 \\ 3 & 9 \end{pmatrix}$$

\therefore Berdasarkan a. (vi)

$$A - (B - C) \neq (A - B) - C$$

Jadi pernyataan N salah

5

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$$

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 18 \\ -6 & 13 \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^2 + aA + bI = 0$$

$$\begin{pmatrix} 1 & 18 \\ -6 & 13 \end{pmatrix} + \begin{pmatrix} 2a & 3a \\ -a & 4a \end{pmatrix} + \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} = 0$$

$$\Rightarrow -6 - a + 0 = 0$$

$$a = -6$$

$$\Rightarrow 1 + 2a + b = 0$$

$$1 + 2(-6) + b = 0$$

$$1 - 12 + b = 0$$

$$b = 11$$

$$\therefore a = -6, b = 11$$

$$\begin{aligned} \text{[6.]} \quad P - K I &= \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} \\ &= \begin{pmatrix} 1-k & 2 \\ 2 & 1-k \end{pmatrix} \end{aligned}$$

$$|P - KI| = 0$$

$$\begin{vmatrix} 1-k & 2 \\ 2 & 1-k \end{vmatrix} = 0$$

$$(1-k)(1-k) - 4 = 0$$

$$1^2 - k - k + k^2 - 4 = 0$$

$$k^2 - 2k - 3 = 0$$

$$(k-3)(k+1) = 0$$

$$k=3 \quad k=-1$$

∴ nilai k adalah 3 dan -1

$$\text{[7]} \quad A = \begin{pmatrix} 2 & 2 \\ 3 & -1 \end{pmatrix}$$

$$a.) A^2 = \begin{pmatrix} 10 & 2 \\ 3 & 7 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 26 & 18 \\ 27 & -1 \end{pmatrix}$$

$$b.) f(x) = x^3 - 3x^2 - 2x + 4$$

$$f(A) = A^3 - 3A^2 - 2A + 4$$

$$= \begin{pmatrix} 26 & 18 \\ 27 & -1 \end{pmatrix} - 3 \begin{pmatrix} 10 & 2 \\ 3 & 7 \end{pmatrix} - 2 \begin{pmatrix} 2 & 2 \\ 3 & -1 \end{pmatrix} +$$

$$= \begin{pmatrix} 26 & 18 \\ 27 & -1 \end{pmatrix} - \begin{pmatrix} 30 & 6 \\ 9 & 21 \end{pmatrix} - \begin{pmatrix} 4 & 4 \\ 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -8 & 8 \\ 12 & -20 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 8 \\ 12 & -16 \end{pmatrix}$$

$$c.) g(x) = x^2 - x - 8$$

$$g(A) = A^2 - A - 8$$

$$= \begin{pmatrix} 10 & 2 \\ 3 & 7 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 3 & -1 \end{pmatrix} - \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\text{[8]} \quad P = \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ 3 & -2 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & -2 & -4 \\ 3 & 0 & 4 \end{pmatrix}$$

$$\begin{aligned} a.) \quad PQ &= \begin{pmatrix} 2-3 & -4 & -8-4 \\ 1+0 & -2 & -4 \\ 3-6 & -6 & -12-8 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -4 & -12 \\ 1 & -2 & -4 \\ -3 & -6 & -20 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} b.) \quad QP &= \begin{pmatrix} 2-2-12 & -1+0+8 \\ 6+0+12 & -3+0-8 \end{pmatrix} \\ &= \begin{pmatrix} -12 & 7 \\ 18 & -11 \end{pmatrix} \end{aligned}$$

$$\boxed{9} \quad M = \begin{pmatrix} 4 & 5 \\ -2 & -1 \end{pmatrix}, \quad N = \begin{pmatrix} -1 & 4 \\ 2 & 0 \end{pmatrix}$$

$$\begin{aligned} a) M^{-1} &= \frac{1}{-4+10} \begin{pmatrix} -1 & -5 \\ 2 & 4 \end{pmatrix} \\ &= \frac{1}{6} \begin{pmatrix} -1 & -5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{6} & -\frac{5}{6} \\ \frac{2}{6} & \frac{4}{6} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} b) N^{-1} &= \frac{1}{0-8} \begin{pmatrix} 0 & -4 \\ -2 & -1 \end{pmatrix} \\ &= -\frac{1}{8} \begin{pmatrix} 0 & -4 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{8} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} c) (MN)^{-1} &= \left(\begin{pmatrix} 4 & 5 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 2 & 0 \end{pmatrix} \right)^{-1} \\ &= \left(\begin{pmatrix} -4+10 & 16 \\ 2-2 & -8 \end{pmatrix} \right)^{-1} \\ &= \begin{pmatrix} 6 & 16 \\ 0 & -8 \end{pmatrix}^{-1} \\ &= \frac{1}{-48} \begin{pmatrix} -8 & -16 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} \\ 0 & -\frac{1}{8} \end{pmatrix} \end{aligned}$$

$$\boxed{10} \quad M = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}, \quad X = \dots ?$$

$$\begin{aligned} a) MX &= \begin{pmatrix} 8 & -4 \\ 5 & -1 \end{pmatrix} \\ X &= \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 8 & -4 \\ 5 & -1 \end{pmatrix} \\ &= \frac{1}{4+2} \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 8 & -4 \\ 5 & -1 \end{pmatrix} \\ &= \frac{1}{6} \begin{pmatrix} 8+10 & -4-2 \\ -8+20 & 4-4 \end{pmatrix} \\ &= \frac{1}{6} \begin{pmatrix} 18 & -6 \\ 12 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} b) X &= \begin{pmatrix} 8 & -10 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 8 & -10 \\ 2 & 2 \end{pmatrix} \frac{1}{6} \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix} \\ &= \frac{1}{6} \begin{pmatrix} 8+10 & 16-40 \\ 2-2 & 4+8 \end{pmatrix} \\ &= \frac{1}{6} \begin{pmatrix} 18 & -24 \\ 0 & 12 \end{pmatrix} \\ &= \begin{pmatrix} 3 & -4 \\ 0 & 2 \end{pmatrix} \end{aligned}$$

$$\boxed{11} \quad P = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{aligned} a) |P| &= 1 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} + 0 \\ &= (1-2) - (2) \\ &= -3 \end{aligned}$$

$$\begin{aligned} b) \text{Adj } P &= \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} \end{pmatrix} \\ &= \begin{pmatrix} -1 & -1 & 1 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \end{aligned}$$

$$c) P^{-1} = \frac{\text{Adj } P}{|P|} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{4}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

12 a)

$$\begin{aligned} X &= \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}^{-1} \begin{pmatrix} -2 & 4 \\ 6 & 0 \end{pmatrix} \\ &= \frac{1}{6-4} \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} -2 & 4 \\ 6 & 0 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} -6-6 & 12 \\ 8+12 & -16 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} -12 & 12 \\ 20 & -16 \end{pmatrix} \\ &= \begin{pmatrix} -6 & 6 \\ 10 & -8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} b.) \quad X &= \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}^{-1} \begin{pmatrix} -4 & 6 \\ -7 & 8 \end{pmatrix} \\ &= \frac{1}{8-3} \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -4 & 6 \\ -7 & 8 \end{pmatrix} \\ &= \frac{1}{5} \begin{pmatrix} -16+21 & 24-24 \\ 4-14 & -6+16 \end{pmatrix} \\ &= \frac{1}{5} \begin{pmatrix} 5 & 0 \\ -10 & 10 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -2 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} c.) \quad X &= \begin{pmatrix} 6 & 5 \\ 1 & 9 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 6 & 5 \\ 1 & 9 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix} \frac{1}{5-6} \\ &= -1 \begin{pmatrix} 6-15 & -12+25 \\ 1-27 & -2+45 \end{pmatrix} \\ &= -1 \begin{pmatrix} -9 & 13 \\ -26 & -43 \end{pmatrix} \\ &= \begin{pmatrix} 9 & -13 \\ 26 & 43 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} d.) \quad X &= \begin{pmatrix} -2 & 4 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 8 & 5 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} -2 & 4 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ -8 & 2 \end{pmatrix} \frac{1}{10-8} \\ &= \frac{1}{2} \begin{pmatrix} -10-32 & 2+8 \\ -48 & 12 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} -42 & 10 \\ -48 & 12 \end{pmatrix} \\ &= \begin{pmatrix} -21 & 5 \\ -24 & 6 \end{pmatrix} \end{aligned}$$

13 a.) $3x + 4y = -11$
 $2x - y = 0$

$$\Rightarrow D = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = -11$$

$$\Rightarrow D_x = \begin{vmatrix} -11 & 4 \\ 0 & -1 \end{vmatrix} = 11$$

$$\Rightarrow D_y = \begin{vmatrix} 3 & -11 \\ 2 & 0 \end{vmatrix} = -22$$

$$\Rightarrow x = \frac{D_x}{D} = \frac{11}{-11} = -1$$

$$y = \frac{D_y}{D} = \frac{-22}{-11} = 2$$

\therefore Himpunan penyelesaiannya $\{(-1, 2)\}$

b.) $-4x - 2y = 22$
 $3x - 5y = -23$

$$D = \begin{vmatrix} -4 & -2 \\ 3 & -5 \end{vmatrix} = 20+6 = 26$$

$$D_x = \begin{vmatrix} 22 & -2 \\ -23 & -5 \end{vmatrix} = -110-46 = -156$$

$$D_y = \begin{vmatrix} -4 & 22 \\ 3 & -23 \end{vmatrix} = 92-66 = 26$$

$$\Rightarrow x = \frac{D_x}{D} = \frac{-156}{26} = -6, \quad y = \frac{D_y}{D} = \frac{26}{26} = 1$$

$\therefore x = -6$
 $y = 1$

12

12) c) $-2x + 3y = 13$
 $x + 5y = 39$

$$\Rightarrow D = \begin{vmatrix} -2 & 3 \\ 1 & 5 \end{vmatrix} = -10 - 3 = -13$$

$$D_x = \begin{vmatrix} 13 & 3 \\ 39 & 5 \end{vmatrix} = 65 - 117 = -52$$

$$D_y = \begin{vmatrix} -2 & 13 \\ 1 & 39 \end{vmatrix} = -78 - 13 = -91$$

$$x = \frac{D_x}{D} = \frac{-52}{-13} = 4$$

$$y = \frac{D_y}{D} = \frac{-91}{-13} = 7$$

$$\therefore (x, y) = (4, 7)$$

d) $12x + y = 0$

$$9x - 8y = 0$$

$$\Rightarrow D = \begin{vmatrix} 12 & 1 \\ 9 & -8 \end{vmatrix} = -96 - 9 = -105$$

$$D_x = \begin{vmatrix} 0 & 1 \\ 0 & -8 \end{vmatrix} = 0$$

$$D_y = \begin{vmatrix} 12 & 0 \\ 9 & 0 \end{vmatrix} = 0$$

$$\therefore (x, y) = (0, 0)$$

e) $x + y + z = 0$

$$2x - y - 4z = 15$$

$$x - 2y - z = 7$$

$$\Rightarrow D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -4 \\ 1 & -2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= 1 - 4 - 4 + 1 - 8 + 2$$

$$= -12$$

$$\Rightarrow D_x = \begin{vmatrix} 0 & 1 & 1 \\ 15 & -1 & -4 \\ 7 & -2 & -1 \end{vmatrix} \begin{vmatrix} 0 & 1 \\ 15 & -1 \\ 7 & -2 \end{vmatrix}$$

$$= 0 - 28 - 30 + 7 - 0 + 15$$

$$= -36$$

$$D_y = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 15 & -4 \\ 1 & 7 & -1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 2 & 15 \\ 1 & 7 \end{vmatrix}$$

$$= -15 + 0 + 14 - 15 + 28 - 0$$

$$= 12$$

$$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 15 \\ 1 & -2 & 7 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= -7 + 15 + 0 - 0 + 30 - 14$$

$$= 24$$

$$x = \frac{D_x}{D} = \frac{-36}{-12} = 3 \quad ; \quad y = \frac{D_y}{D} = \frac{12}{-12} = -1$$

$$z = \frac{D_z}{D} = \frac{24}{-12} = -2 \quad \therefore (x, y, z) = (3, -1, -2)$$

f) $x - 5y + 3z = 12$

$$2x + 3y - 6z = 3$$

$$3x - 4y + 9z = 25$$

$$\Rightarrow D = \begin{vmatrix} 1 & -5 & 3 \\ 2 & 3 & -6 \\ 3 & -4 & 9 \end{vmatrix} \begin{vmatrix} 1 & -5 \\ 2 & 3 \\ 3 & -4 \end{vmatrix}$$

$$= 27 + 90 - 24 - 27 - 24 + 90$$

$$= 132$$

$$D_x = \begin{vmatrix} 12 & -5 & 3 \\ 3 & 3 & -6 \\ 25 & -4 & 9 \end{vmatrix} \begin{vmatrix} 12 & -5 \\ 3 & 3 \\ 25 & -4 \end{vmatrix}$$

$$= 324 + 750 - 36 - 225 - 288 + 135$$

$$= 660$$

$$D_y = \begin{vmatrix} 1 & 12 & 3 \\ 2 & 3 & -6 \\ 3 & 25 & 9 \end{vmatrix} \begin{vmatrix} 1 & 12 \\ 2 & 3 \\ 3 & 25 \end{vmatrix}$$

$$= 27 - 216 + 150 - 27 + 150 - 216$$

$$= -132$$

$$\Rightarrow D_z = \begin{vmatrix} 1 & -5 & 12 \\ 2 & 3 & 3 \\ 3 & -4 & 25 \end{vmatrix} \begin{vmatrix} 1 & -5 \\ 2 & 3 \\ 3 & -4 \end{vmatrix}$$

$$= 75 - 45 - 96 - 108 + 12 + 250$$

$$= 88$$

$$x = \frac{D_x}{D} = \frac{660}{132} = 5$$

$$z = \frac{D_z}{D} = \frac{88}{132} = \frac{2}{3}$$

$$y = \frac{D_y}{D} = \frac{-132}{132} = -1$$

$$\therefore (x, y, z) = \left(5, -1, \frac{2}{3} \right)$$

9.) $3x - 2y + 5z = 6$

$$4x - 4y + 3z = 0$$

$$5x - 4y + z = -5$$

$$\Rightarrow D = \begin{vmatrix} 3 & -2 & 5 \\ 4 & -4 & 3 \\ 5 & -4 & 1 \end{vmatrix} \begin{vmatrix} 6 & -2 \\ 4 & -4 \\ 5 & -4 \end{vmatrix}$$

$$= -12 - 30 - 80 + 100 + 36 + 8$$

$$= 22$$

$$D_x = \begin{vmatrix} 6 & -2 & 5 \\ 0 & -4 & 3 \\ -5 & -4 & 1 \end{vmatrix} \begin{vmatrix} 6 & -2 \\ 0 & -4 \\ -5 & -4 \end{vmatrix}$$

$$= -24 + 30 + 0 - 100 + 72 + 0$$

$$= -22$$

$$D_y = \begin{vmatrix} 3 & 6 & 5 \\ 4 & 0 & 3 \\ 5 & -5 & 1 \end{vmatrix} \begin{vmatrix} 3 & 6 \\ 4 & 0 \\ 5 & -5 \end{vmatrix}$$

$$= 0 + 90 - 100 - 0 + 45 - 24$$

$$= 11$$

$$D_z = \begin{vmatrix} 3 & -2 & 6 \\ 4 & -4 & 0 \\ 5 & -4 & -5 \end{vmatrix} \begin{vmatrix} 3 & -2 \\ 4 & -4 \\ 5 & -4 \end{vmatrix}$$

$$= 60 + 0 - 96 + 120 - 0 - 40$$

$$= 44$$

$$\Rightarrow x = \frac{D_x}{D} = \frac{-22}{22} = -1$$

$$z = \frac{D_z}{D} = \frac{44}{22} = 2$$

$$y = \frac{D_y}{D} = \frac{11}{22} = \frac{1}{2}$$

$$\therefore (x, y, z) = \left(-1, \frac{1}{2}, 2 \right)$$

14) misal Panjang = x
lebar = y

$$\Rightarrow 2x + 2y = 420$$

$$x = y + 30$$

$$x - y = 30$$

$$\begin{cases} 2x + 2y = 420 \\ x - y = 30 \end{cases}$$

$$a) \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 420 \\ 30 \end{pmatrix}$$

$$b) D = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = -2 - 2 = -4$$

$$D_x = \begin{vmatrix} 420 & 2 \\ 30 & -1 \end{vmatrix} = -420 - 60 = -480$$

$$D_y = \begin{vmatrix} 2 & 420 \\ 1 & 30 \end{vmatrix} = 60 - 420 = -360$$

$$x = \frac{D_x}{D} = \frac{-480}{-4} = 120$$

$$y = \frac{D_y}{D} = \frac{-360}{-4} = 90$$

\therefore Panjangnya adalah 120 cm dan
lebar nya 90 cm

15) Ali $\rightarrow 5x + 3y = 27.000$
 Ani $\rightarrow 4x + 2y = 21.000$

a) $\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 27.000 \\ 21.000 \end{pmatrix}$

b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 27.000 \\ 21.000 \end{pmatrix}$
 $= \frac{1}{10-12} \begin{pmatrix} 2 & -3 \\ -4 & 5 \end{pmatrix} \begin{pmatrix} 27.000 \\ 21.000 \end{pmatrix}$
 $= \frac{1}{-2} \begin{pmatrix} 54.000 - 63.000 \\ -108.000 + 105.000 \end{pmatrix}$
 $= \frac{-1}{2} \begin{pmatrix} -9.000 \\ -3.000 \end{pmatrix}$
 $= \begin{pmatrix} 4.500 \\ 1.500 \end{pmatrix}$

\therefore Harga bukutulis = $x = 4.500$ dan bolpoin adalah $y = 1.500$

16. a) $5x + 4y = 241.000$
 $10x + 6y = 434.000$

b) $\begin{pmatrix} 5 & 4 \\ 10 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 241.000 \\ 434.000 \end{pmatrix}$
 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 10 & 6 \end{pmatrix}^{-1} \begin{pmatrix} 241.000 \\ 434.000 \end{pmatrix}$
 $= \frac{1}{30-40} \begin{pmatrix} 6 & -4 \\ -10 & 5 \end{pmatrix} \begin{pmatrix} 241.000 \\ 434.000 \end{pmatrix}$
 $= \frac{-1}{10} \begin{pmatrix} 1.446.000 - 1.736.000 \\ -2.410.000 + 2.170.000 \end{pmatrix}$
 $= \frac{-1}{10} \begin{pmatrix} -290.000 \\ -240.000 \end{pmatrix}$
 $= \begin{pmatrix} 29.000 \\ 24.000 \end{pmatrix}$

c) Buku matematika = $x = 29.000$
 Buku Informatika = $y = 24.000$

17) matriks awal

$$\begin{pmatrix} 600 & 800 & 700 \\ 400 & 500 & 200 \end{pmatrix}$$

matriks setelah terjadi kenaikan 20%

$$\begin{pmatrix} 720 & 960 & 840 \\ 480 & 600 & 240 \end{pmatrix}$$

18) $P = \begin{pmatrix} 0 & 20 \\ 10 & 15 \\ 20 & 10 \end{pmatrix}$

$$Q = \begin{pmatrix} 600 & 800 & 700 \\ 400 & 500 & 200 \end{pmatrix}$$

a) $P \cdot Q = \begin{matrix} & \text{lagu} & \text{pesi} & \text{buku} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{pmatrix} 8000 & 10.000 & 4000 \\ 12.000 & 15.500 & 10.000 \\ 16.000 & 21.000 & 16.000 \end{pmatrix} \end{matrix}$

b) $\det(PQ) = 0$

maka matriks inversenya tidak dapat ditentukan.

19) F $\rightarrow x + 3y + 2z = 380.000 \dots (i)$

S $\rightarrow 5y + 2z = 520.000 \dots (ii)$

$3x = z$

$3x - z = 0 \dots (iii)$

a) $\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 2 \\ 3 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 380.000 \\ 520.000 \\ 0 \end{pmatrix}$

b) $D = \begin{vmatrix} 1 & 3 & 2 \\ 0 & 5 & 2 \\ 3 & 0 & -1 \end{vmatrix} \begin{vmatrix} 1 & 3 \\ 0 & 5 \\ 3 & 0 \end{vmatrix}$

$= -5 + 18 + 0 - 30 - 0 - 0$

$= -17$

$$\Rightarrow D_x = \begin{vmatrix} 380.000 & 3 & 2 \\ 520.000 & 5 & 2 \\ 0 & 0 & -1 \end{vmatrix} \begin{matrix} 380.000 \\ 520.000 \\ 0 \end{matrix}$$

$$= -1.900.000 + 0 + 0 + 1.560.000$$

$$= -340.000$$

$$D_y = \begin{vmatrix} 1 & 380.000 & 2 \\ 0 & 520.000 & 2 \\ 3 & 0 & -1 \end{vmatrix} \begin{matrix} 1 \\ 0 \\ 3 \end{matrix} \begin{matrix} 380.000 \\ 520.000 \\ 0 \end{matrix}$$

$$= -520.000 + 2.280.000 - 3.120.000$$

$$= -1.360.000$$

$$D_z = \begin{vmatrix} 1 & 3 & 380.000 \\ 0 & 5 & 520.000 \\ 3 & 0 & 0 \end{vmatrix} \begin{matrix} 1 \\ 0 \\ 3 \end{matrix} \begin{matrix} 380.000 \\ 520.000 \\ 0 \end{matrix}$$

$$= 4.680.000 - 5.700.000$$

$$= -1.020.000$$

$$x = \frac{D_x}{D} = \frac{-340.000}{-17} = 20.000$$

$$y = \frac{D_y}{D} = \frac{-1.360.000}{-17} = 80.000$$

$$z = \frac{D_z}{D} = \frac{-1.020.000}{-17} = 60.000$$

∴ Harga mi goreng = 20.000, Sate rusa = 80.000,

dan nasi goreng = 60.000

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-9} \begin{pmatrix} 1 & -4 & 2 \\ -8 & 5 & 2 \\ 2 & 1 & -5 \end{pmatrix} \begin{pmatrix} 820.000 \\ 800.000 \\ 650.000 \end{pmatrix}$$

$$= \frac{1}{-9} \begin{pmatrix} -1.080.000 \\ -1.260.000 \\ -810.000 \end{pmatrix}$$

$$= \begin{pmatrix} 120.000 \\ 140.000 \\ 90.000 \end{pmatrix}$$

∴ Harga Satu Kemeja adalah 120.000,

harga satu celana adalah 140.000

dan harga satu kaus adalah 90.000

$$\boxed{20} \quad 3x + 2y + 2z = 820.000$$

$$4x + y + 2z = 800.000$$

$$2x + y + 3z = 650.000$$

$$a) \begin{pmatrix} 3 & 2 & 2 \\ 4 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 820.000 \\ 800.000 \\ 650.000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 \\ 4 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}^{-1} \begin{pmatrix} 820.000 \\ 800.000 \\ 650.000 \end{pmatrix}$$