

Uji Capaian Pembelajaran 2
A. Pilihan Ganda.

$$1. \frac{2(\cos 150^\circ - \sin 120^\circ)}{\sin 30^\circ}$$

$$2. \frac{2(\cos(180^\circ - 30^\circ) - \sin(180^\circ - 60^\circ))}{\frac{1}{2}}$$

$$= 4(-\cos 30^\circ - \sin 60^\circ)$$

$$= 4(-\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{3})$$

$$= -4\sqrt{3}$$

Jawaban: A

$$2. \tan(90^\circ - x)^\circ = \frac{\sin(90^\circ - x)^\circ}{\cos(90^\circ - x)^\circ}$$

$$= \frac{\cos x^\circ}{\sin x^\circ}$$

Karena $\sin x^\circ = m \Rightarrow \sin^2 x = m^2$

$$\Rightarrow 1 - \sin^2 x = 1 - m^2$$

$$\Rightarrow \cos^2 x = 1 - m^2$$

$$\Rightarrow \cos x = \sqrt{1 - m^2}$$

$$\therefore \tan(90^\circ - x)^\circ = \frac{\sqrt{1 - m^2}}{m}$$

Jawaban: D

$$3. \frac{\sin 30^\circ \tan 135^\circ \cos 45^\circ}{\cos 60^\circ \sin 225^\circ \tan 150^\circ}$$

$$= \frac{\frac{1}{2} \cdot \tan(180^\circ - 45^\circ) \cdot \frac{1}{2}\sqrt{2}}{\frac{1}{2} \cdot \sin(180^\circ + 45^\circ) \cdot \tan(180^\circ - 30^\circ)}$$

$$= \frac{(-\frac{1}{2}) \cdot \frac{1}{2}\sqrt{2} \cdot 1}{(-\frac{1}{2}\sqrt{2}) \cdot (-\frac{1}{3}\sqrt{3})} = -\frac{3}{\sqrt{3}} = -\sqrt{3}$$

Jawaban: B.

$$4. \frac{\sin^4 x - \cos^4 x}{\tan^2 x - 1}$$

$$= \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}}$$

$$= \frac{(\sin^2 x - \cos^2 x) \cdot 1}{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}} = \cos^2 x$$

Jawaban: A

$$5. \frac{1 - \sin^2 A}{2 \cos^2 A} = \frac{1 \cdot \cos^2 A}{2 \cdot \cos^2 A} = \frac{1}{2}$$

Jawaban: C

$$6. \cos^2 A (1 + \tan^2 A) = \cos^2 A (\sec^2 A)$$

$$= 1$$

$$= \frac{\cos^2 A}{1 - \sin A} - \frac{\cos^2 A}{1 + \sin A}$$

$$= \cos^2 A \left(\frac{1}{1 - \sin A} - \frac{1}{1 + \sin A} \right)$$

$$= \cos^2 A \cdot \frac{1 + \sin A - (1 - \sin A)}{1 - \sin^2 A}$$

$$= \cancel{\cos^2 A} \cdot \frac{2 \sin A}{\cancel{\cos^2 A}} = 2 \sin A$$

$$\circ \text{ Jelas } \sec^2 A = \tan^2 A + 1$$

$$\circ 4 \cos^2 A + 4 \sin^2 A = 4(\cos^2 A + \sin^2 A)$$

$$= 4$$

$$\circ (\sin A + \cos A)(\sin A - \cos A)$$

$$= \sin^2 A - \cos^2 A \neq 1$$

\therefore Identitas trigonometri yang tidak benar adalah

$$(\sin A + \cos A)(\sin A - \cos A) = 1$$

Jawaban: E.

Uji Capaian Pembelajaran 2

7. $\tan A = \frac{1}{4}$

$$\frac{\sin A + \cos A}{3 \cos A - 2 \sin A}$$

$$= \frac{\sin A + \cos A}{3 \cos A - 2 \sin A} \cdot \frac{\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\sin A}{\cos A} + 1}{3 \cdot 1 - 2 \cdot \frac{\sin A}{\cos A}}$$

$$= \frac{\tan A + 1}{3 - 2 \cdot \tan A} = \frac{\frac{1}{4} + 1}{3 - 2 \cdot \frac{1}{4}} = \frac{\frac{5}{4}}{\frac{10}{4}} = \frac{1}{2}$$

Jawaban: C.

8. Fungsi trigonometri (FT)

$$\equiv y = A \sin(bx + c)$$

• Maksimum = 2 ; minimum = -2

$$\Rightarrow FT \equiv y = 2 \sin(bx + c)$$

• Jarak (nilai x) titik puncak minimum ke titik potong sumbu x

$$= \frac{4}{3} \pi - \frac{5}{6} \pi = \frac{8}{6} \pi - \frac{5}{6} \pi = \frac{3}{6} \pi = \frac{1}{2} \pi$$

$$\Rightarrow 1 \text{ gelombang penuh} = 4 \cdot \frac{1}{2} \pi = 2 \pi = 360^\circ$$

$$\Rightarrow FT \equiv y = 2 \sin(x + c)$$

• Nilai x yang membuat nilai minimum untuk $\sin x = \frac{3}{2} \pi$

$$\Rightarrow \text{selisih} = \frac{3}{2} \pi - \frac{4}{3} \pi = \frac{1}{6} \pi$$

$$\therefore FT \equiv y = 2 \sin(x + \frac{\pi}{6})$$

Jawaban: A.

9. Fungsi trigonometri (FT)

$$\equiv y = a \sin(bx + c)$$

• Maksimum = 2 ; minimum = -2

$$\Rightarrow FT \equiv y = 2 \sin(bx + c)$$

• $(0, \sqrt{2})$ ada di kurva FT

$$\Rightarrow -\sqrt{2} = 2 \cdot \sin c$$

$$\Leftrightarrow \sin c = -\frac{1}{2} \sqrt{2} \Leftrightarrow c = -45^\circ$$

$$\Rightarrow FT \equiv y = 2 \sin(bx - 45^\circ)$$

• Selisih absis titik minimum dan absis titik maksimum

$$= 105^\circ - 45^\circ = 60^\circ = \frac{1}{2} \text{ gelombang}$$

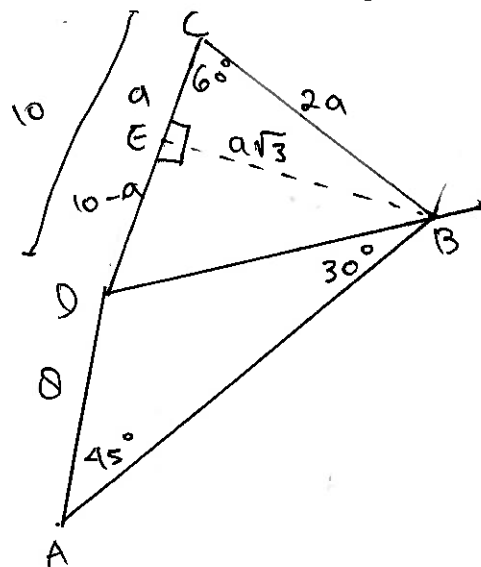
$$\Rightarrow 1 \text{ gelombang} = 2 \cdot 60^\circ = 120^\circ = \frac{1}{3} \cdot 360^\circ$$

$$\Rightarrow FT = 2 \sin(3x - 45^\circ)$$

Jawaban: C.

10. ilustrasi segi empat ABCD

(gambar ada kemungkinan tidak sesuai bentuknya).



• Aturan sinus di ABD

$$\frac{DB}{\sin 45^\circ} = \frac{DA}{\sin 30^\circ}$$

$$\Rightarrow DB = \frac{DA}{\sin 30^\circ} \cdot \sin 45^\circ$$

$$= \frac{8}{\frac{1}{2}} \cdot \frac{\sqrt{2}}{2} = 8\sqrt{2}$$

Uji Capaian Pembelajaran 2

- Buat titik bantuan E sehingga $CE \perp EB$ dan $E \in CD$; $CE = a$

$$\Rightarrow ED \perp BE; BE = a\sqrt{3}; BC = 2a$$

$$\Rightarrow BD^2 = BE^2 + ED^2$$

$$\Leftrightarrow (8\sqrt{2})^2 = (a\sqrt{3})^2 + (10-a)^2$$

$$\Leftrightarrow 128 = 3a^2 + a^2 - 20a + 100$$

$$\Leftrightarrow 4 \cdot 32 = 4 \cdot a^2 - 20a + 4 \cdot 25$$

$$\Leftrightarrow 4 \cdot a^2 - 4 \cdot 5a - 4 \cdot 7 = 0$$

$$\Leftrightarrow a^2 - 5a - 7 = 0$$

$$\Rightarrow a = \frac{5 + \sqrt{5^2 - 4 \cdot 1 \cdot (-7)}}{2 \cdot 1}$$

$$\Rightarrow BC = 2a = 5 + \sqrt{53}$$

Jawaban: E

$$11. \angle A = 30^\circ; \angle C = 60^\circ \Rightarrow \angle B = 90^\circ$$

- Misalkan $BC = a$

$$\Rightarrow AB = a\sqrt{3}, AC = 2a$$

$$\circ AC + BC = 15$$

$$\Leftrightarrow 2a + a = 15 \Leftrightarrow 3a = 15 \Leftrightarrow a = 5$$

$$\Rightarrow AB = 5\sqrt{3} \text{ cm}$$

Jawaban: E

$$12. p = 10 \text{ cm}; r = 4 \text{ cm}; \angle Q = 60^\circ$$

- Aturan cosinus $\triangle PQR$ di $\angle Q$

$$\Leftrightarrow q^2 = p^2 + r^2 - 2 \cdot p \cdot r \cdot \cos \angle Q$$

$$= 10^2 + 4^2 - 2 \cdot 10 \cdot 4 \cdot \cos 60^\circ$$

$$= 116 - 2 \cdot 40 \cdot \frac{1}{2}$$

$$= 76$$

$$\Rightarrow q = \sqrt{76} = \sqrt{4 \cdot 19} = 2\sqrt{19}$$

Jawaban: B.

$$13. QR = 3\sqrt{6}; PR = 6\sqrt{2}; PQ = 3\sqrt{2}$$

- Aturan cosinus $\triangle PQR$ di $\angle P$

$$\Leftrightarrow QR^2 = PR^2 + PQ^2 - 2 \cdot PR \cdot PQ \cdot \cos \angle P$$

$$\Leftrightarrow (3\sqrt{6})^2 = (6\sqrt{2})^2 + (3\sqrt{2})^2$$

$$- 2 \cdot 6\sqrt{2} \cdot 3\sqrt{2} \cdot \cos \angle P$$

$$\Leftrightarrow 54 = 72 + 18 - 72 \cdot \cos \angle P$$

$$\Leftrightarrow 72 \cos \angle P = 36$$

$$\Leftrightarrow \cos \angle P = \frac{1}{2} \Rightarrow \angle P = 60^\circ$$

Jawaban: C.

$$14. KM = 7; LM = 8; \angle LMK = 120^\circ$$

- Aturan cosinus $\triangle KLM$ di $\angle LMK (LM)$

$$\Leftrightarrow KL^2 = KM^2 + LM^2 - 2 \cdot KM \cdot LM \cos \angle LM$$

$$= 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cdot \cos 120^\circ$$

$$= 113 - 2 \cdot 56 \cdot \left(-\frac{1}{2}\right)$$

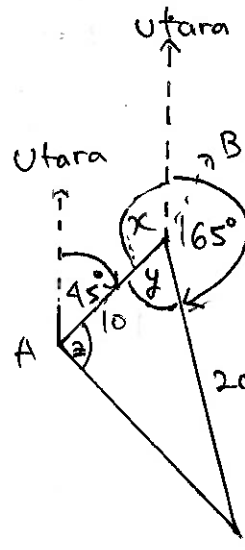
$$= 169$$

$$\Rightarrow KL = \sqrt{169} = 13 \text{ cm}$$

$$\therefore \text{keliling } \triangle KLM = 7 + 8 + 13 = 28$$

Jawaban: C.

15.



$$x = 180^\circ - 45^\circ = 135^\circ$$

$$y = 360^\circ - (165^\circ + x) \\ = 360^\circ - (165^\circ + 135^\circ) \\ = 60^\circ$$

$$\circ AB : BC = 10 : 20 \\ = 1 : 2$$

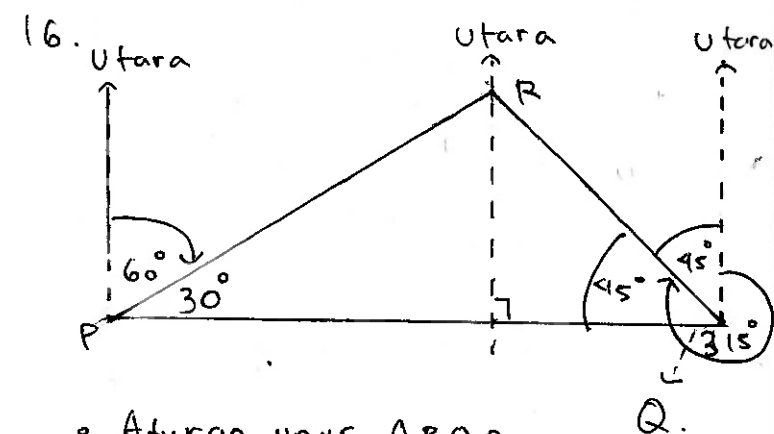
- Perhatikan. $\angle ABC = 60^\circ$

- Perbandingan sisi untuk segitiga dengan sudut $30^\circ, 60^\circ, 90^\circ$

$$= 1 : \sqrt{3} : 2$$

$$\Rightarrow AC = 10\sqrt{3} \Rightarrow \text{Jawaban: D.}$$

Uji Capaian Pembelajaran 2



• Aturan sinus ΔPQR .

$$\frac{PR}{\sin \angle Q} = \frac{QR}{\sin \angle P}$$

$$\Rightarrow PR = \frac{\sin \angle Q}{\sin \angle P} \cdot QR = \frac{\sin 45^\circ}{\sin 30^\circ} \cdot QR$$

$$= \frac{\frac{1}{2}\sqrt{2}}{\frac{1}{2}} \cdot QR = QR\sqrt{2}$$

• Misalkan waktu tempuh kedua kapal = t

$$\Rightarrow V_A = \frac{PR}{t} \text{ dan } V_B = \frac{QR}{t}$$

$$\Rightarrow V_A = \frac{PR}{t} = \frac{QR\sqrt{2}}{t} = V_B\sqrt{2}$$

$\therefore V_A$ harus $\sqrt{2}$ kali $V_B \Rightarrow x = \sqrt{2}$.

Jawaban: D.

17. $\angle P = 30^\circ$; $\angle Q = (20^\circ) \Rightarrow \angle R = 30^\circ$

$$\Rightarrow QR = PQ = 9$$

$$\therefore \text{Luas } \Delta PQR = \frac{1}{2} \cdot PQ \cdot QR \cdot \sin \angle Q$$

$$= \frac{1}{2} \cdot 9 \cdot 9 \cdot \sin 120^\circ$$

$$= \frac{1}{2} \cdot 81 \cdot \frac{1}{2}\sqrt{3}$$

$$= 20,25\sqrt{3} \text{ cm}^2$$

Jawaban: E.

18. $\angle CAB = 45^\circ$, $AC = 8$; $AB = 5$

$$\Rightarrow \text{Luas } \Delta ABC = \frac{1}{2} \cdot AC \cdot AB \cdot \sin \angle CAB$$

$$= \frac{1}{2} \cdot 8 \cdot 5 \cdot \frac{1}{2}\sqrt{2}$$

$$= 10\sqrt{2}$$

• ABCD = jajargenjang

$$\Rightarrow \Delta BCD = \Delta ABC$$

$$\Rightarrow \text{Luas ABCD} = 2 \times \text{Luas } \Delta ABC$$

$$= 2 \times 10\sqrt{2} = 20\sqrt{2}$$

Jawaban: D

19. Segi enam dengan sisi = 6

\Rightarrow Pada segi enam tersebut terdapat 6 buah segitiga identik dengan

• 1 sudut dengan besar

$$= \frac{360^\circ}{6} = 60^\circ, \text{ dan}$$

• 2 sudut dengan besar

$$= \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

\Rightarrow segitiga yang dimaksud adalah segitiga sama sisi

\Rightarrow Luas segi enam dengan sisi 6 cm

$$= 6 \times \text{Luas segitiga sama sisi dengan sisi 6 cm}$$

$$= 6 \times \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin 60^\circ$$

$$= 3 \cdot 6 \cdot 3 \cdot \frac{1}{2}\sqrt{3}$$

$$= 54\sqrt{3}$$

Jawaban: C.

Uji Capaian Pembelajaran 2

20. Karena

$$\bullet 2 \geq 1 \Rightarrow f(2) = 6$$

$$\Leftrightarrow 2a(2) - 2b = 6$$

$$\Leftrightarrow 2a - b = 3 \dots \text{Pers (1)}$$

$$\bullet -3 < 1 \Rightarrow f(-3) = -4$$

$$\Leftrightarrow -3a + b = -4$$

$$\bullet \text{Pers (1)} \Rightarrow \frac{2a - b = 3}{-a = -1 \Leftrightarrow a = 1}$$

$$\bullet \text{Pers (1)} \Rightarrow 2a - b = 3$$

$$a = 1 \Rightarrow 2(1) - b = 3$$

$$\Leftrightarrow b = -1$$

$$\therefore a + b = 1 + (-1) = 0$$

Jawaban: B

$$21. f(x) = \begin{cases} 3x + a, & \text{jika } x < -1 \\ 2x - 1, & \text{jika } x \geq -1 \end{cases}$$

$f(x)$ akan kontinu jika

$$3(-1) + a = 2(-1) - 1$$

$$\Leftrightarrow -3 + a = -3 \Leftrightarrow a = 0$$

Jawaban: C.

22. Fungsi Eksponensial (FE)

$$\equiv f(x) = a^{x+b} + c$$

• Asimtot datar FE $\equiv y = 1$

$$\Rightarrow FE \equiv f(x) = a^{x+b} + 1$$

• $(0, 2)$ melalui kurva FE

$$\Rightarrow 2 = a^b + 1 \Leftrightarrow a^b = 1$$

Karena FE adalah fungsi eksponensial $\Rightarrow b = 0$

$$\bullet \Rightarrow FE \equiv f(x) = a^x + 1$$

• $(1, 3)$ melalui kurva FE

$$\Rightarrow 3 = a^1 + 1 \Leftrightarrow a^1 = 2 \Leftrightarrow a = 2$$

$$\Rightarrow FE \equiv f(x) = 2^x + 1$$

Jawaban: C

23. Asimtot yang ada pada kurva yaitu asimtot tegak $x = 0$

\therefore Jawaban yang memungkinkan harus berbentuk fungsi logaritma

Jawaban: E.

$$24. f(x) = \frac{7 \log x}{1 - 2 \cdot 7 \log x} =$$

$$\begin{aligned} \Rightarrow f\left(\frac{1}{x}\right) &= \frac{7 \log\left(\frac{1}{x}\right)}{1 - 2 \cdot 7 \log\left(\frac{1}{x}\right)} \\ &= \frac{7 \log 7 - 7 \log x}{1 - 2 \cdot (7 \log 7 - 7 \log x)} \\ &= \frac{1 - 7 \log x}{1 - 2(1 - 7 \log x)} \\ &= \frac{1 - 7 \log x}{1 - 2 + 2 \cdot 7 \log x} \\ &= \frac{1 - 7 \log x}{-1 + 2 \cdot 7 \log x} \\ &= \frac{7 \log x - 1}{1 - 2 \cdot 7 \log x} \end{aligned}$$

$$\begin{aligned} \therefore f\left(\frac{1}{x}\right) + f(x) &= \frac{7 \log x - 1}{1 - 2 \cdot 7 \log x} \\ &\quad + \frac{7 \log x}{1 - 2 \cdot 7 \log x} \\ &= \frac{2 \cdot 7 \log x - 1}{-(2 \cdot 7 \log x - 1)} \\ &= -1 \end{aligned}$$

Jawaban: B

Uji Capaian Pembelajaran 2

$$25. a \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} - 2b \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \\ 10 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a - 2b \\ a + 2b - 3c \\ 3a + 5c \end{pmatrix} = \begin{pmatrix} -7 \\ 0 \\ 10 \end{pmatrix}$$

$$\bullet a - 2b = -7 \Leftrightarrow 2b = a + 7$$

$$\bullet a + 2b - 3c = 0 \Leftrightarrow a + (a + 7) - 3c = 0 \\ \Leftrightarrow 2a - 3c = -7.$$

$$\bullet \begin{array}{l|l} 2a - 3c = -7 & \times 3 \\ 3a + 5c = 10 & \times 2 \end{array} \Rightarrow \begin{array}{l} 6a - 9c = -21 \\ 6a + 10c = 20 \\ \hline -19c = -41 \\ \Rightarrow c = 3 \end{array}$$

$$\bullet 2a - 3c = -7 \Rightarrow 2a - 3(3) = -7 \\ \Leftrightarrow 2a = 2 \Leftrightarrow a = 1$$

$$\bullet 2b = a + 7 = 1 + 7 = 8$$

$$\Rightarrow b = \frac{8}{2} = 4 \Leftrightarrow b = 4.$$

$$\therefore a + b + c = 1 + 4 + 3 = 8.$$

Jawaban: B.

$$26. \vec{P} = (1 \ 2\sqrt{2} \ \sqrt{3})$$

$$\vec{Q} = (1 \ 2\sqrt{2} \ -\sqrt{3})$$

$$\Rightarrow |\vec{P}| = \sqrt{1^2 + (2\sqrt{2})^2 + (\sqrt{3})^2} \\ = \sqrt{1 + 8 + 3} = \sqrt{12}$$

$$\Rightarrow |\vec{Q}| = \sqrt{1^2 + (2\sqrt{2})^2 + (-\sqrt{3})^2} \\ = \sqrt{1 + 8 + 3} = \sqrt{12}.$$

$$\Rightarrow \vec{P} \cdot \vec{Q} = 1 \cdot 1 + (2\sqrt{2}) \cdot (2\sqrt{2}) \\ + (\sqrt{3}) \cdot (-\sqrt{3}) \\ = 1 + 8 - 3 = 6$$

$$\bullet \cos \alpha = \frac{\vec{P} \cdot \vec{Q}}{|\vec{P}| |\vec{Q}|} = \frac{6}{\sqrt{12} \cdot \sqrt{12}} = \frac{6}{12} = \frac{1}{2}$$

$$\alpha = 60^\circ$$

Jawaban: C

$$27. \vec{PQ} = \begin{pmatrix} 2-1 \\ 3-(-1) \\ 3-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

$$\vec{PR} = \begin{pmatrix} -1-1 \\ 2-(-1) \\ 1-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

\therefore Panjang proyeksi orthogonal

vektor \vec{PQ} pada \vec{PR}

$$= \frac{\vec{PQ} \cdot \vec{PR}}{|\vec{PR}|} = \frac{1 \cdot (-2) + 4 \cdot 3 + 3 \cdot 1}{\sqrt{(-2)^2 + 3^2 + 1^2}} \\ = \frac{13}{\sqrt{14}} = \frac{13}{14} \sqrt{14}$$

Jawaban: A.

$$28. \vec{P} = 2\vec{i} - \vec{j} + 2\vec{k} ; \vec{Q} = 3\vec{i} - 6\vec{j} + 3\vec{k}$$

\bullet Proyeksi Orthogonal vektor \vec{Q} pada \vec{P}

$$= \frac{\vec{P} \cdot \vec{Q}}{|\vec{P}|^2} \cdot \vec{P} \\ = \frac{2 \cdot 3 + (-1) \cdot (-6) + 2 \cdot 3}{2^2 + (-1)^2 + 2^2} (2\vec{i} - \vec{j} + 2\vec{k}) \\ = \frac{18}{9} (2\vec{i} - \vec{j} + 2\vec{k}) \\ = 2(2\vec{i} - \vec{j} + 2\vec{k}) \\ = 4\vec{i} - 2\vec{j} + 4\vec{k}$$

Jawaban: D.

Uji Capaian Pembelajaran

29. $\vec{a} = (p, 1, 3)$, $\vec{b} = (10, q, 15)$

\vec{a} dan \vec{b} segaris

$$\Rightarrow \vec{b} = k \cdot \vec{a}$$

$$\Leftrightarrow (10, q, 15) = k(p, 1, 3)$$

$$\Leftrightarrow (10, q, 15) = (kp, k, 3k)$$

$$\Rightarrow 3k = 15 \Leftrightarrow k = 5$$

$$\circ q = k = 5 \Leftrightarrow q = 5$$

$$\circ 10 = kp = 3p \Leftrightarrow p = \frac{10}{3} = 2$$

$$\Leftrightarrow p = 2$$

a Pernyataan 1 (benar)

$$p = 2, q = 5$$

• Pernyataan 2 (salah)

vektor \vec{a} dan \vec{b} harus membentuk sudut 0° bukan 90°

• Pernyataan 3 (salah).

Panjang vektor \vec{a} dan \vec{b} tidak harus sama

• Pernyataan 4 (benar)

$$\vec{b} = 5 \cdot \vec{a} \Rightarrow |\vec{b}| = |5 \cdot \vec{a}| = 5 |\vec{a}|$$

$$\Rightarrow |\vec{b}| = 5 \cdot |\vec{a}|$$

\therefore Pernyataan yang benar adalah pernyataan (i) dan (iv)

Jawaban: C.

30. $AD:DB = 2:1$; $D \in AB$

o misalkan $D(a, b, c)$.

$$\Rightarrow \vec{AD} = 2 \vec{DB}$$

$$\Rightarrow \begin{cases} \circ a - 1 = 2(4 - a) \Leftrightarrow 3a = 9 \Leftrightarrow a = 3 \\ \circ b - (-1) = 2(5 - b) \Leftrightarrow 3b = 9 \Leftrightarrow b = 3 \\ \circ c - 2 = 2(2 - c) \Leftrightarrow 3c = 6 \Leftrightarrow c = 2 \end{cases}$$

$$\therefore D(3, 3, 2).$$

$$\vec{CD} = (3 - 1, 3 - 0, 2 - 4)$$

$$\Rightarrow \vec{CD} = (2, 3, -2)$$

\therefore Panjang CD

$$\begin{aligned} = |\vec{CD}| &= \sqrt{2^2 + 3^2 + (-2)^2} \\ &= \sqrt{4 + 9 + 4} \\ &= \sqrt{17} \end{aligned}$$

Jawaban: A.

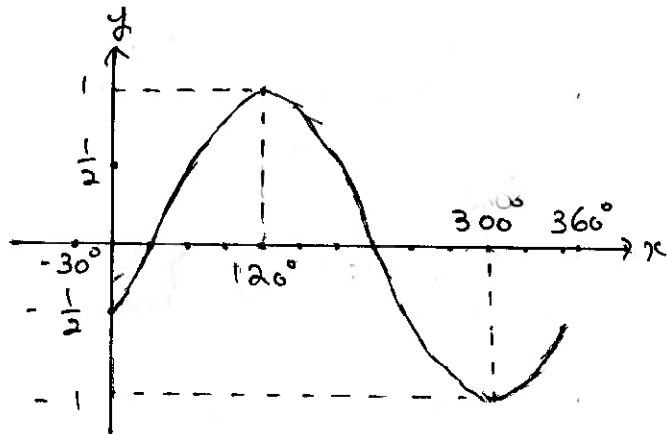
Uji Capaian Pembelajaran 2

B. Uraian

1. a). $y = \sin(x - 30^\circ) \equiv \text{kurva } C_1$

$y = \sin x \equiv \text{kurva } C_2$

\therefore kurva C_1 merupakan hasil penggeseran kurva C_2 sejauh 30° (panjang) ke arah kanan

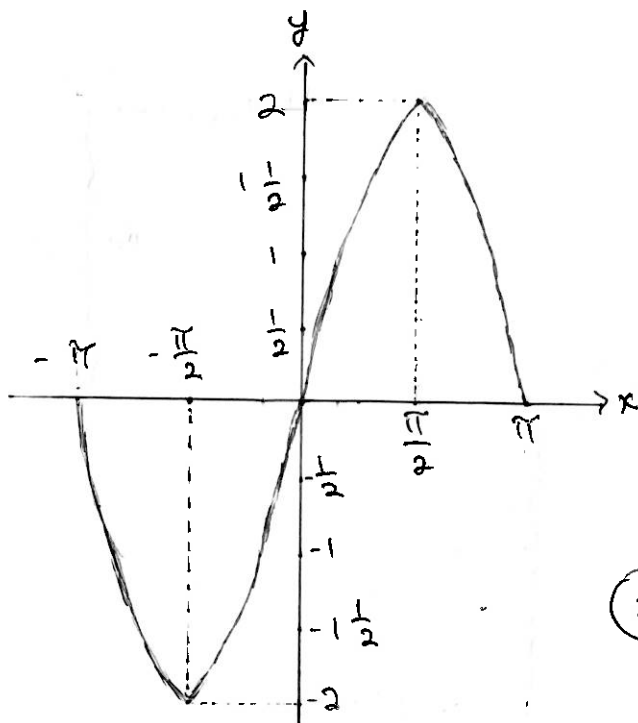


b). kurva $C_1 \equiv y = 2 \cos(x - \frac{\pi}{2})$

kurva $C_2 \equiv y = 2 \cos x$

kurva $C_3 \equiv y = \cos x$

\therefore kurva C_1 merupakan kurva C_3 yang mengalami pembesaran 2 kali dan penggeseran ke arah kanan sebesar $\frac{\pi}{2}$.



$$2. a). \frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \cdot \frac{\sin A}{\cos A}}{\sec^2 A}$$

$$= \frac{2 \sin A}{\cos A} \times \cos^2 A$$

$$= 2 \sin A \cos A$$

\therefore Terbukti $\frac{2 \tan A}{1 + \tan^2 A} = 2 \sin A \cos A$

b).

$$(1 - \sin^2 A) \tan^2 A = \cos^2 A \tan^2 A$$

$$= \cos^2 A \cdot \frac{\sin^2 A}{\cos^2 A}$$

$$= 1 - \cos^2 A$$

$\therefore (1 - \sin^2 A) \tan^2 A = 1 - \cos^2 A$

c).

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A}$$

$$= \frac{\sin A (1 - \cos A)}{(1 + \cos A)(1 - \cos A)} + \frac{(1 + \cos A) \sin A}{\sin A \cdot \sin A}$$

$$= \frac{\sin A - \sin A \cos A}{1 - \cos^2 A} + \frac{\sin A + \sin A \cos A}{\sin^2 A}$$

$$= \frac{\sin A - \cancel{\sin A \cos A}}{\sin^2 A} + \frac{\sin A + \cancel{\sin A \cos A}}{\sin^2 A}$$

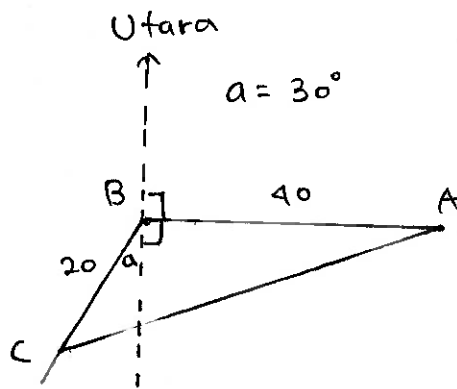
$$= \frac{2 \sin A}{\sin^2 A} = \frac{2}{\sin A} = 2 \csc A$$

$\therefore \frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \csc A$

3. • Kota A di sebelah timur kota B dengan jarak 40 km

\equiv kota A berada pada arah 090° dari kota B dengan jarak 40 km.

• Kota C berada pada arah 210° dari kota B dengan jarak 20 km.



$$\angle ABC = 210^\circ - 90^\circ = 120^\circ$$

Jarak kota A dan kota C = AC

• Aturan Cosinus $\triangle ABC$ di $\angle B$

$$\begin{aligned} AC^2 &= AB^2 + CB^2 - 2 \cdot AB \cdot CB \cdot \cos \angle B \\ &= 40^2 + 20^2 - 2 \cdot 40 \cdot 20 \cdot \cos 120^\circ \\ &= 2.000 - 2 \cdot 800 \left(-\frac{1}{2}\right) \\ &= 2.800 \end{aligned}$$

$$\Rightarrow AC = \sqrt{2.800} = \sqrt{400 \cdot 7} = 20\sqrt{7} \text{ km}$$

$$\therefore \text{Jarak kota A dan kota C} = 20\sqrt{7} \text{ km}$$

4. a). kurva memiliki asimtot datar

\Rightarrow kurva tersebut merupakan kurva fungsi eksponensial (FE)

$$\bullet FE \equiv f(x) = a^x + b$$

$$\bullet \text{Asimtot datar FE} \equiv y = 0$$

$$\Rightarrow FE \equiv f(x) = a^x$$

$$\bullet (-2, 9) \text{ ada di kurva}$$

$$\Rightarrow 9 = a^{-2} \Leftrightarrow \left(\frac{1}{3}\right)^{-2} = a^{-2}$$

$$\Leftrightarrow a = \frac{1}{3}$$

$$\therefore FE \equiv f(x) = \left(\frac{1}{3}\right)^x$$

b). Kurva memiliki asimtot tegak

\Rightarrow kurva tersebut adalah kurva fungsi logaritma (FL)

$$\bullet FL \equiv f(x) = {}^a \log (bx - c) + d$$

$$\bullet \text{Asimtot tegak FL} \equiv x = 1$$

$$\Rightarrow c = 1 \text{ dan } d = 0$$

$$\Rightarrow FL \equiv f(x) = {}^a \log (bx - 1)$$

$$\bullet (2, 0) \text{ ada di kurva FL}$$

$$\Rightarrow 0 = {}^a \log (2b - 1)$$

$$\Rightarrow 2b - 1 = 1$$

$$\Leftrightarrow 2b = 2$$

$$\Leftrightarrow b = 1$$

$$\therefore \Rightarrow FL \equiv f(x) = {}^a \log (x - 1)$$

$$\bullet (5, 2) \text{ ada di kurva FL}$$

$$\Rightarrow 2 = {}^a \log (5 - 1) = {}^a \log 4 = {}^a \log 2^2$$

$$\Rightarrow a = 2$$

$$\therefore FL \equiv f(x) = {}^2 \log (x - 1)$$

c. Kurva memiliki asimtot datar $y = 2$

\Rightarrow kurva berbentuk kurva fungsi eksponensial (FE)

$$\bullet FE \equiv f(x) = a \cdot b^x + 2$$

$$\bullet (0, 4) \text{ ada di kurva FE}$$

$$\Rightarrow 4 = a + 2 \Leftrightarrow a = 2$$

$$\Rightarrow FE \equiv f(x) = 2 \cdot b^x + 2$$

$$\bullet (1, 3) \text{ ada di kurva FE}$$

$$\Rightarrow 3 = 2 \cdot b^1 + 2$$

$$\Rightarrow 1 = 2b \Leftrightarrow b = \frac{1}{2}$$

$$\Rightarrow FE \equiv f(x) = 2 \cdot \left(\frac{1}{2}\right)^x + 2$$

Uji Capaian Pembelajaran 2

$$5. \vec{a} = \begin{pmatrix} 2 \\ -6 \\ -3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$|\vec{a}| = \sqrt{2^2 + (-6)^2 + (-3)^2}$$

$$= \sqrt{4 + 36 + 9}$$

$$= \sqrt{49}$$

$$= 7$$

$$|\vec{b}| = \sqrt{2^2 + 1^2 + (-2)^2}$$

$$= \sqrt{4 + 1 + 4}$$

$$= \sqrt{9}$$

$$= 3$$

$$\vec{a} \cdot \vec{b} = 2 \cdot 2 + (-6) \cdot 1 + (-3) \cdot (-2)$$

$$= 4 - 6 + 6$$

$$= 4$$

a). (i) Proyeksi skalar orthogonal dari

vektor \vec{a} pada arah vektor \vec{b}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{4}{3}$$

(ii) Proyeksi skalar orthogonal dari

vektor \vec{b} pada arah vektor \vec{a}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{4}{7}$$

b). (i) Proyeksi vektor orthogonal dari

vektor \vec{a} pada arah vektor \vec{b}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b} = \frac{4}{3^2} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \frac{4}{9} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{9} \\ \frac{4}{9} \\ -\frac{8}{9} \end{pmatrix}$$

(ii) Proyeksi vektor orthogonal dari
vektor \vec{b} pada arah vektor \vec{a}

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a} = \frac{4}{7^2} \cdot \begin{pmatrix} 2 \\ -6 \\ -3 \end{pmatrix} = \frac{4}{49} \begin{pmatrix} 2 \\ -6 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{8}{49} \\ -\frac{24}{49} \\ -\frac{12}{49} \end{pmatrix}$$

Soal Model Ak M Halaman 368

$$G_1(x) = A_1 \sin k_1 x$$

$$G_2(x) = A_2 \sin k_2 x$$

$$G_3(x) = A_3 \sin k_3 x$$

• titik maksimum $G_1 = (15^\circ, 4)$

⇒ 1 gelombang = $4 \cdot 15^\circ = 60^\circ = \frac{1}{6} \cdot 360^\circ$

⇒ $G_1(x) = 4 \sin 6x$

• Titik maksimum $G_2 = (30^\circ, 6)$

⇒ 1 gelombang = $4 \cdot 30^\circ = 120^\circ = \frac{1}{3} \cdot 360^\circ$

⇒ $G_2(x) = 6 \sin 3x$

• Titik maksimum $G_3 = (45^\circ, 5)$

⇒ 1 gelombang = $4 \cdot 45^\circ = 180^\circ = \frac{1}{2} \cdot 360^\circ$

⇒ $G_3(x) = 5 \sin 2x$

1. Tarif = Rp 3.000,00/km

• Pernyataan 1

$$G_1(5^\circ) = 4 \cdot \sin(6 \cdot 5^\circ) = 4 \cdot \sin 30^\circ$$

$$= 4 \cdot \frac{1}{2} = 2 \text{ km}$$

∴ Penghasilan $G_1(5^\circ) = 2 \times 3.000$
 $= 6.000$

• Pernyataan 2

Penghasilan dengan $G_2(10^\circ)$

$$= 3.000 \times (6 \cdot \sin(3 \cdot 10^\circ))$$

$$= 3.000 \times (6 \cdot \frac{1}{2})$$

$$= 9.000$$

• Pernyataan 3

Penghasilan dengan $G_3(15^\circ)$

$$= 3.000 \times (5 \cdot \sin(2 \cdot 15^\circ))$$

$$= 3.000 \times (5 \cdot \frac{1}{2})$$

$$= 7.500$$

• Pernyataan 4

• Penghasilan dengan $G_1(10^\circ)$

$$= 3.000 \times (4 \cdot \sin(6 \cdot 10^\circ))$$

$$= 3.000 \times (4 \cdot \sin 60^\circ)$$

$$= 3.000 \times 4 \cdot \frac{1}{2} \sqrt{3}$$

$$= 6.000 \times 1,7$$

$$= 10.200$$

• Penghasilan dengan $G_2(10^\circ)$

$$= 3.000 \times (6 \cdot \sin(3 \cdot 10^\circ))$$

$$= 9.000$$

∴ selisih penghasilan $G_1(10^\circ)$ dan

$$G_2(10^\circ)$$

$$= 10.200 - 9.000 = 1.200$$

• Pernyataan 5

• Penghasilan dengan $G_2(30^\circ)$

$$= 3.000 \times (6 \cdot \sin(3 \cdot 30^\circ))$$

$$= 3.000 \times (6 \cdot \sin 90^\circ)$$

$$= 3.000 \times 6$$

$$= 18.000$$

• Penghasilan dengan $G_3(30^\circ)$

$$= 3.000 \times (5 \cdot \sin(2 \cdot 30^\circ))$$

$$= 3.000 \times (5 \cdot \sin 60^\circ)$$

$$= 3.000 \times (5 \cdot \frac{1}{2} \sqrt{3})$$

$$= 7.500 \sqrt{3}$$

$$= 7.500 \times 1,7$$

$$= 12.750$$

∴ selisih penghasilan $G_2(30^\circ)$ dan

$$G_3(30^\circ)$$

$$= 18.000 - 12.750$$

$$= 5.250$$

2.0 Pernyataan 1 (Benar)

$$\text{Tarif} = \text{Rp } 2.000,00/\text{km}.$$

$$G_2(15^\circ) = 6 \sin(3 \cdot 15^\circ)$$

$$= 6 \sin(45^\circ)$$

$$= 6 \cdot \frac{1}{2} \sqrt{2}$$

$$= 3 \cdot 1,4$$

$$= 4,2 \text{ km}$$

$$G_3(15^\circ) = 5 \sin(2 \cdot 15^\circ)$$

$$= 5 \cdot \sin 30^\circ$$

$$= 5 \cdot \frac{1}{2}$$

$$= 2,5 \text{ km}.$$

\therefore selisih tarif ojol 2 dan 3 ketika berjalan ke arah 15°

$$= 2.000 \times (4,2 - 2,5)$$

$$= 2.000 \times 1,7$$

$$= 3.400$$

o Pernyataan 2 (Salah).

$$\text{Tarif} = \text{Rp } 2.500,00/\text{km}.$$

$$G_3(15^\circ) = 5 \cdot \sin(2 \cdot 15^\circ) = 2,5 \text{ km}.$$

$$G_1(15^\circ) = 4 \cdot \sin(6 \cdot 15^\circ)$$

$$= 4 \cdot \sin 90^\circ$$

$$= 4 \text{ km}.$$

\therefore selisih tarif ojol 1 dan 3 ketika berjalan ke arah 15°

$$= 2.500 (4 - 2,5)$$

$$= 2.500 \cdot 1,5$$

$$= 3.750 \neq 2.750$$

o Pernyataan 3 (Benar)

$$G_1(10^\circ) = 4 \cdot \sin(6 \cdot 10^\circ) =$$

$$= 4 \cdot \sin 60^\circ$$

$$= 4 \cdot \frac{1}{2} \sqrt{3}$$

$$= 2 \cdot 1,7$$

$$= 3,4 \text{ km}$$

$$G_1(20^\circ) = 4 \cdot \sin(6 \cdot 20^\circ)$$

$$= 4 \cdot \sin 120^\circ$$

$$= 4 \cdot \frac{1}{2} \sqrt{3}$$

$$= 3,4 \text{ km}$$

karena $G_1(20^\circ) = 3,4 \text{ km} = G_1(10^\circ)$.

\therefore Besar penghasilan ojol 1 sama besar ketika berjalan arah 10° dan 20°

o Pernyataan 4 (Benar)

$$G_2(10^\circ) = 6 \cdot \sin(3 \cdot 10^\circ)$$

$$= 6 \cdot \sin 30^\circ$$

$$= 6 \cdot \frac{1}{2}$$

$$= 3 \text{ km}$$

$$G_2(50^\circ) = 6 \cdot \sin(3 \cdot 50^\circ)$$

$$= 6 \cdot \sin 150^\circ$$

$$= 6 \cdot \frac{1}{2}$$

$$= 3 \text{ km}$$

karena $G_2(10^\circ) = G_2(50^\circ)$

\Rightarrow Besar penghasilan ojol 2 sama besar ketika berjalan dengan arah 10° dan 50° .

• Pernyataan 5

$$\begin{aligned} G_3(15^\circ) &= 5 \cdot \sin(2 \cdot 15^\circ) \\ &= 5 \cdot \sin 30^\circ \\ &= 5 \cdot \frac{1}{2} \\ &= 2,5 \text{ km} \end{aligned}$$

$$\begin{aligned} G_3(75^\circ) &= 5 \cdot \sin(2 \cdot 75^\circ) \\ &= 5 \cdot \sin 150^\circ \\ &= 5 \cdot \frac{1}{2} \\ &= 2,5 \text{ km} \end{aligned}$$

karena $G_3(15^\circ) = G_3(75^\circ) = 2,5 \text{ km}$

\Rightarrow Besar penghasilan pengemudi ojol 3
 sama besar ketika berjalan dengan
 arah 15° dan 75°

$$\begin{aligned} 3. \quad G_2(45^\circ) &= 6 \cdot \sin(3 \cdot 45^\circ) \\ &= 6 \cdot \sin 135^\circ \\ &= 6 \cdot \frac{1}{2} \sqrt{2} \\ &= 3 \cdot 1,4 \\ &= 4,2 \text{ km} \end{aligned}$$

$$\begin{aligned} G_3(45^\circ) &= 5 \cdot \sin(2 \cdot 45^\circ) \\ &= 5 \cdot \sin 90^\circ \\ &= 5 \text{ km} \end{aligned}$$

tarif = Rp 3.000,00 / km

\Rightarrow Jumlah penghasilan pengemudi
 ojol 2 dan 3 ketika berjalan
 dengan arah 45°

$$\begin{aligned} &= 3.000 \times (4,2 + 5) \\ &= 3.000 \times 9,2 \\ &= 27.600 \end{aligned}$$

Jawaban: C.